Exercises for

Chapter 3: Data Representation

1 EXERCISE 1

Consider the following datasets:

- The evolution in time of the prices of N different stock-exchange shares, recorded at one-second intervals over the period of one hour.
- The paths covered by all cars driving through a given city, recorded at one-minute intervals over the period of one hour. For each record, we store the car ID, the car's position, and the car's speed.
- The amount of rainfall and the air temperature, recorded at a given time instant at N given weather stations over some geographical area.

Describe the kind of grid, grid cells, and data attributes that you would use to store such a dataset. Argue your proposal by considering the kind of data to store, and the locations at which data is recorded (sampled).

2 EXERCISE 2

Sampling and reconstruction are closely related operations which reduce a function $y = f(x)$ to a finite set of sample points (x_i, y_i) and, respectively, reconstruct an approximation $\tilde{y} = \tilde{f}(x)$ of $f(x)$ from the sample points. Consider an application where you have to perform the above reconstruction $\tilde{f}(x)$, but you are only allowed to use a fixed finite number N of sample points x_i. How would you place these sample points over the domain of definition of x so that the reconstruction error $\|\tilde{f} - f\|$ is equally well minimized over the entire range of x?

Hints: first, consider the kinds of basis functions you want to use (*e.g.*, constant or linear). Next, consider how you can minimize the reconstruction error by shifting the points x_i around the x axis.

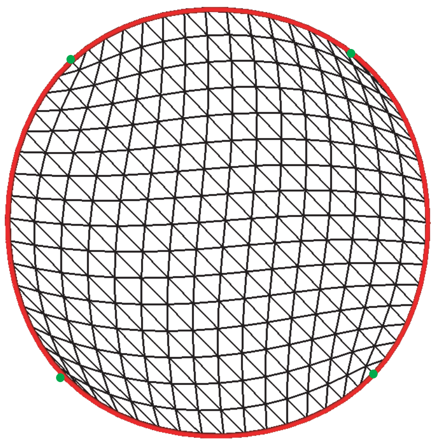
3 EXERCISE 3

In Figure 3.10 in Chapter 3 (also displayed below), it is shown that we can use structured grids to cover a 2D disk shape. Now, consider an arbitrary convex 2D shape of genus 0 (that is, without holes). The 2D shape is specified by means of its contour, which is given as a closed 2D polyline of N points.

- Can we always construct a structured grid so that all points of this polyline will be also points on the grid's boundary? If not, sketch a simple counter-example.

- Can we always construct a structured grid with the conditions listed in the point above *and* the additional condition that no grid-boundary point exists which is not a polyline point? If not, sketch a simple counter-example.

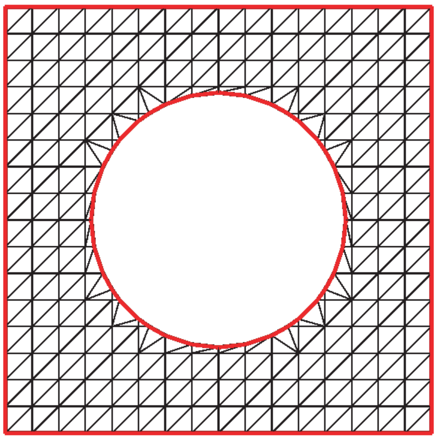
Hints: Think about the number of points on the boundary of a structured grid.



A 2D structured grid covering a disk shape. The grid corners are indicated in green (see Chapter 3).

4 EXERCISE 4

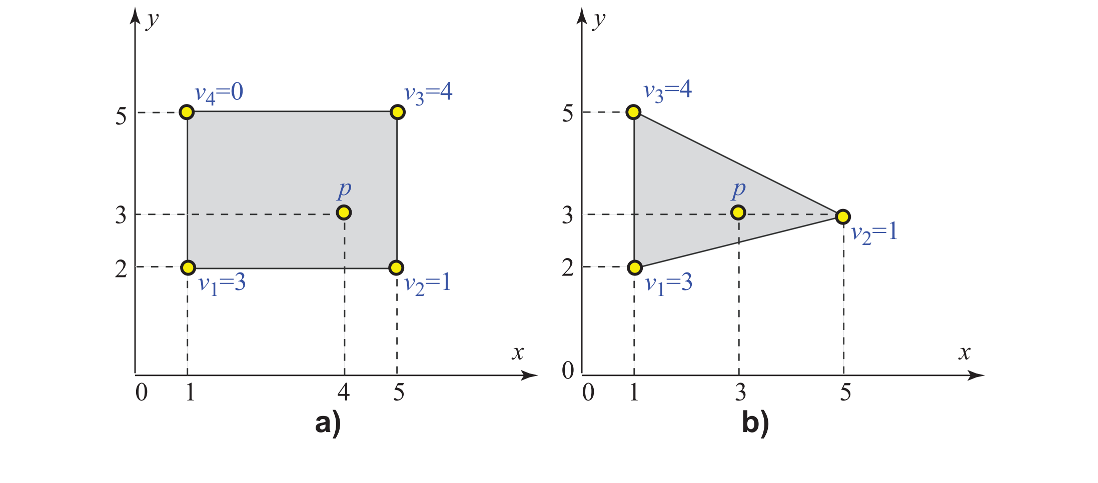
As shown in Figure 3.11 in Chapter 3 (also shown below), not all 2D shapes can be covered by structured grids. Consider now a 3D (curved) surface of a half sphere. Can we cover this surface with a structured grid? Argue your answer.



A 2D genus-1 shape can only be covered by an unstructured grid (see Chapter 3).

5 EXERCISE 5

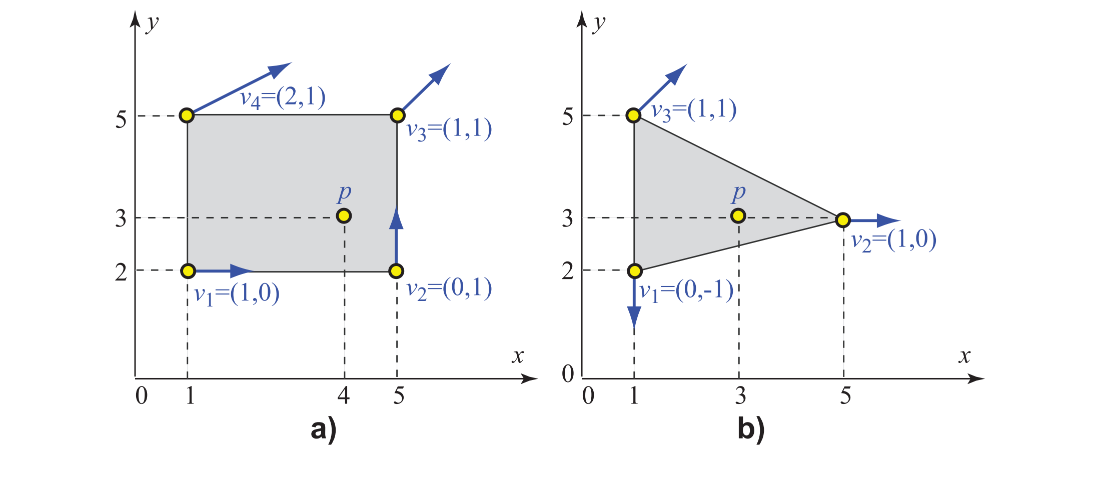
Consider the 2D cells in the figure below. For each cell, scalar data values v_i are indicated at its sample points (vertices). Additionally, a separate point p inside the cell is indicated. If bilinear interpolation is used, compute the interpolated value $v(p)$ of the vertex data values v_i at the point p. Detail your answer by explaining how you computed the interpolated value.



Cell interpolation questions for scalar data.

6 EXERCISE 6

Consider the 2D cells in the figures below. For each cell, vector data values $\mathbf{v}_i$ are indicated at its sample points (vertices). Additionally, a separate point p inside the cell is indicated. If bilinear interpolation is used, compute the interpolated value $\mathbf{v}(p)$ of the vertex data values $\mathbf{v}_i$ at the point p. Detail your answer by explaining how you computed the interpolated value.



Cell interpolation questions for vector data.

7 EXERCISE 7

Color selection, by end users, is typically done by various widgets which represent the space of available colors, such as the color wheel, color hexagon, or three separate color sliders for the R, G, and B (or alternatively H, S, and V) color components. Assume, now, that we want to select only colors present in a given *subset* of the entire color space. Concretely, we have a large set of color photographs, and we next want to select only colors predominantly present in these photographs, rather than any possible color. Sketch and argue for a color-selection widget that would optimally help users to select only these specific colors.

Hints: Think how to modify any of the existing color-selection widgets to 'focus' on a specific color range where many samples exist.

8 EXERCISE 8

Consider a grid where we have color data values recorded at its cell vertices. We would like to use linear interpolation to compute colors at all points inside the grid cells. We can do this by interpolating colors represented as RGB triplets or, alternatively, colors represented as HSV triplets. Discuss the advantages and disadvantages of both schemes. Can you imagine a situation where the RGB interpolation would be arguably preferable to HSV interpolation? Can you imagine a situation when the converse (HSV interpolation is preferable to RGB interpolation) is true? Describe such situations or alternatively argue for the fact that they do not exist.

9 EXERCISE 9

Consider a grid cell, such as a 1D line, 2D triangle or quad, or 3D parallelepiped or cube, and some scalar values v_i recorded at the cell vertices. Consider that we are using linear interpolation to reconstruct the sampled scalar signal $v(x)$ at any point x inside the cell. Does a cell shape exist, and a point x in that cell, so that $v(x)$ is larger than the maximum of v_i over all cell vertices? Does a cell shape exist, and a point x in that cell, so that $v(x)$ is smaller than the minimum of v_i over all cell vertices? Argue your answers.

10 EXERCISE 10

Consider a grid-cell like in the Exercise 9, and some color values v_i recorded at the cell vertices. Consider that we are using linear interpolation to compute a color $v(x)$ at any point x inside the cell. Does a point x exist so that $v(x)$ is brighter than any of the colors v_i? Does a point x exist so that $v(x)$ is darker than any of the colors v_i? Do the answers to the above two sub-questions depend on the choice of the system, or space, used to represent colors (RGB or HSV)? Explain your answer.

11 EXERCISE 11

Consider the curvature tensor described in Section 3.6.4, *i.e.*, the rate of variation of the curvature of a surface at a point and in a given direction tangent to that surface. Now, consider the surface of a cube. What is the direction of minimal curvature for a point x located on an edge of the cube? What is the direction of maximal curvature for the same point x? Support your answer, if needed, with a sketch.

12 EXERCISE 12

Prove that, for any point x on a spherical surface, the curvature at x in any direction tangent to the sphere, is the same.

Hints: You can either use the analytic definition of curvature as given by the partial derivative of a function describing the sphere-surface in a given direction, or think of its geometrical interpretation as being the curvature of a curve passing through x, located on the surface, and being tangent to the given direction.

13 EXERCISE 13

As discussed in Chapter 3, basis (or interpolation) functions are used to reconstruct a continuous function between a set of sample points. Several types of basis functions are known, such as constant or linear. Can you imagine a dataset where, for each sample point, one would have multiple attributes, and for some attributes we would like to use one type of basis function and, for some other attribute, we would like to use another type of basis function? If so, give an example of such a dataset and argue the decision. If not, argue why this is not advisable.

14 EXERCISE 14

Consider some sampled dataset (x_i, f_i) of some function $f(x)$, in which x belongs to some continuous spatial domain, such as the 2D plane, but f belongs to some domain which is inherently discontinuous, such as a categorical data-range containing the values $\{male, female, animal, object\}$. Given an arbitrary point x in the continuous variable domain, which kind of basis function, from piecewise-constant, linear, or radial, would you argue that is best suitable to interpolate $f(x)$ at that point?

Hints: Think of how you would best mark the value of a point x between some points x_1 and x_2 whose values are some completely unrelated values, e.g. $f(x_1) = female$ and $f(x_2) = animal$.

End of Exercises for
Chapter 3: Data Representation