We introduce a novel method to convert trimmed NURBS surfaces to untrimmed subdivision surfaces with Bézier edge conditions. We take a NURBS surface and its trimming curves as input, from this we automatically compute a base mesh, the limit surface of which fits the trimmed NURBS surface to a specified tolerance. We first construct the topology of the base mesh by performing a cross-field based decomposition in parametric space. The number and positions of extraordinary vertices required to represent the trimmed shape can be automatically identified by smoothing a cross field bounded by the parametric trimming curves. After the topology construction, the control point positions in the base mesh are calculated based on the limit stencils of the subdivision scheme and constraints to achieve tangential continuity across the boundary. Our method can provide the user with either an editable base mesh or a fine mesh whose limit surface approximates the input within a certain tolerance. By integrating the trimming curve as part of the desired limit surface boundary, our conversion can produce gap-free models. Moreover, since we use tangential continuity across the boundary between adjacent surfaces as constraints, the converted surfaces join with $G^1$ continuity.

Context

Non-Uniform Rational B-Splines (NURBS) are the standard freeform surface representation in Computer-Aided Design (CAD) applications. Due to their limitation of a strict rectangular topology, trimming is an important operation to create complex objects. However, it introduces unavoidable gaps when stitching two trimmed NURBS patches together [SFL*08]. Moreover, as the trimmed NURBS are only visually trimmed by skipping the evaluation of the trimmed part in parameter space (Figure 1(a)), the rectangular NURBS topology is not altered although the geometric shape has changed. As a consequence, many operations on trimmed shapes may require a time-consuming re-evaluation process of the trimming curves [Far01], e.g. editing and deformation.

Subdivision, owing to its ability to handle arbitrary topology and its ease of use, has become an attractive alternative to NURBS, especially for modelling in high-end animation [DKT98, Sta98]. Subdivision representations have the great advantage that if two subdivision surfaces share a boundary edge in base meshes, they both contain exactly that piece of boundary curve. This makes it possible to apply exactly the same trimming curves on two intersecting surfaces and thus provide gap-free models.

Although there has been considerable progress in NURBS-compatible subdivision [SZSS98, SZBN03, MRF06, MFR*10, CADS09, CDS09], there is little work addressing conversion between trimmed NURBS surfaces and subdivision surfaces. Conversion from Catmull-Clark subdivision surfaces [CC78] to NURBS surfaces can be achieved by viewing each quadrilateral face of the base mesh as a NURBS patch [LS08]. However, conversion in the other direction is nontrivial. Such conversion is demanded, for example, in industry scenarios when the designer first converts a subdivision surface to NURBS patches and then performs trimming operations to get the desired shape (since trimming NURBS models is well developed in modelling software). The challenge is how to automatically convert the trimmed shape back to the subdivision representation.

The key motivation behind our work is to develop a framework that automatically converts trimmed NURBS surfaces to untrimmed subdivision surfaces. Our aims in conversion are to keep the approximation error within a specified tolerance and to maintain the original continuity across the boundary of adjacent surfaces.

Summary of our method

Given a NURBS surface and a set of trimming curves, the target is to represent the region of the surface within the trimming curves as an untrimmed Catmull-Clark subdivision surface (see Figure 1). We have developed a two-step algorithm to achieve the conversion. We first construct the topology of the base quad mesh for the desired subdivision surface and then calculate the control point positions in the base
References


Further details

The work outlined in this extended abstract appears as a full paper in Computer Aided Geometric Design. DOI: 10.1016/j.cagd.2014.06.004
Figure 2: Conversion results. The first row shows different trimming loops and the corresponding decomposition in the trimmed domain region with 5-valent extraordinary vertices marked in red, 3-valent extraordinary vertices in blue. The second and third rows show the converted subdivision surfaces with two different NURBS surfaces ($S_1$ and $S_2$) as input surface. Note that they use the same decomposition in parameter space (the first row). $S_2$ has high curvature.

Figure 3: Distance error plots of conversion results in Figure 2. The error is scaled by the bounding box diagonal of the trimmed surface. The number of refinement steps is set to three to keep the approximation error below $10^{-4}$. The histograms in the last column explicitly illustrate the error distribution of the examples in the fifth column.