

Conversion of trimmed NURBS surfaces to untrimmed Catmull-Clark subdivision surfaces

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We introduce a novel method to convert trimmed NURBS surfaces to untrimmed subdivision surfaces with Bézier edge conditions. We take a NURBS surface and its trimming curves as input, from this we automatically compute a base mesh, the limit surface of which fits the trimmed NURBS surface to a specified tolerance. We first construct the topology of the base mesh by performing a cross-field based decomposition in parametric space. The number and positions of extraordinary vertices required to represent the trimmed shape can be automatically identified by smoothing a cross field bounded by the parametric trimming curves. After the topology construction, the control point positions in the base mesh are calculated based on the limit stencils of the subdivision scheme and constraints to achieve tangential continuity across the boundary. Our method can provide the user with either an editable base mesh or a fine mesh whose limit surface approximates the input within a certain tolerance. By integrating the trimming curve as part of the desired limit surface boundary, our conversion can produce gap-free models. Moreover, since we use tangential continuity across the boundary between adjacent surfaces as constraints, the converted surfaces join with G^1 continuity.

Context

Non-Uniform Rational B-Splines (NURBS) are the standard freeform surface representation in Computer-Aided Design (CAD) applications. Due to their limitation of a strict rectangular topology, trimming is an important operation to create complex objects. However, it introduces unavoidable gaps when stitching two trimmed NURBS patches together [SFL*08]. Moreover, as the trimmed NURBS are only visually trimmed by skipping the evaluation of the trimmed part in parameter space (Figure 1(a)), the rectangular NURBS topology is not altered although the geometric shape has changed. As a consequence, many operations on trimmed shapes may require a time-consuming re-evaluation process of the trimming curves [Far01], e.g. editing and deformation.

Subdivision, owing to its ability to handle arbitrary topol-

ogy and its ease of use, has become an attractive alternative to NURBS, especially for modelling in high-end animation [DKT98, Sta98]. Subdivision representations have the great advantage that if two subdivision surfaces share a boundary edge in base meshes, they both contain exactly that piece of boundary curve. This makes it possible to apply exactly the same trimming curves on two intersecting surfaces and thus provide gap-free models.

Although there has been considerable progress in NURBS-compatible subdivision [SZSS98, SZBN03, MRF06, MFR*10, CADs09, CDS09], there is little work addressing conversion between trimmed NURBS surfaces and subdivision surfaces. Conversion from Catmull-Clark subdivision surfaces [CC78] to NURBS surfaces can be achieved by viewing each quadrilateral face of the base mesh as a NURBS patch [LS08]. However, conversion in the other direction is nontrivial. Such conversion is demanded, for example, in industry scenarios when the designer first converts a subdivision surface to NURBS patches and then performs trimming operations to get the desired shape (since trimming NURBS models is well developed in modelling software). The challenge is how to automatically convert the trimmed shape back to the subdivision representation.

The key motivation behind our work is to develop a framework that automatically converts trimmed NURBS surfaces to untrimmed subdivision surfaces. Our aims in conversion are to keep the approximation error within a specified tolerance and to maintain the original continuity across the boundary of adjacent surfaces.

Summary of our method

Given a NURBS surface and a set of trimming curves, the target is to represent the region of the surface within the trimming curves as an untrimmed Catmull-Clark subdivision surface (see Figure 1). We have developed a two-step algorithm to achieve the conversion. We first construct the topology of the base quad mesh for the desired subdivision surface and then calculate the control point positions in the base

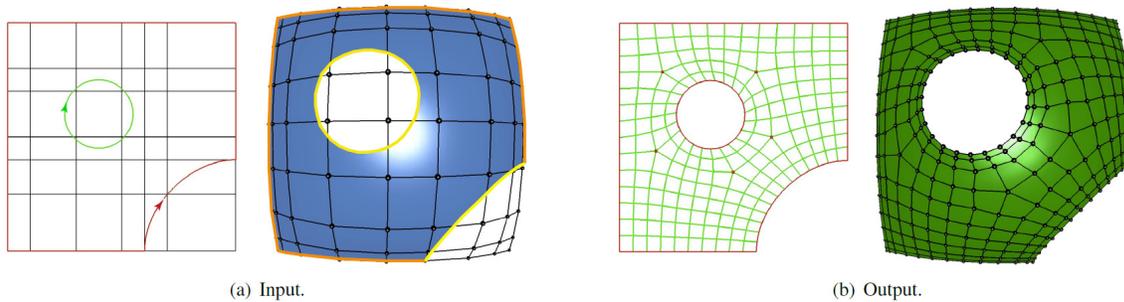


Figure 1: (a) Input. Left: the domain space with black knot lines and oriented trimming loops. Right: the rectangular control mesh of a trimmed NURBS surface. The original NURBS boundary curves marked in orange and the model-space trimming curves marked in yellow comprise the final boundary of the target limit surface. (b) Output. Left: the partition in domain space. Right: the resulting base mesh for Catmull-Clark subdivision [CC78].

quad mesh using the limit stencils of the subdivision scheme. Inspired by the cross field theory in recent quad remeshing techniques [BZK09], we automatically identify the extraordinary vertices required in the base quad mesh and decompose the trimmed domain region into quads by computing a boundary aligned cross field in parametric space. The control point positions in the base quad mesh are chosen to make their corresponding limit points lie exactly on the input NURBS surface. In order to maintain up to G^2 continuity, Bézier edge conditions are applied when constructing the boundary of the base quad mesh. Results can be seen in Figures 2 and 3.

Advantages of the method

The advantages of our conversion are threefold. First, by specifying the number of refinement steps in the topology construction step, we are able to provide the user with either a fully editable control mesh to manipulate the trimmed surface, or a fine control mesh whose limit surface approximates the input trimmed surface with high accuracy. Second, as an advantage of subdivision representation, we can provide gap-free models after conversion. The converted Catmull-Clark surfaces of two intersecting NURBS surfaces will use the same trimming curve as their shared boundary. Third, the original continuity across the boundary of two trimmed surfaces (tangentially intersecting) can be maintained by setting the near-boundary layers of control points to satisfy Bézier edge conditions on both sides.

References

[BZK09] BOMMES D., ZIMMER H., KOBELT L.: Mixed-integer quadrangulation. *ACM Trans. Graphics* 28, 3 (July 2009), 77. 2

[CAD09] CASHMAN T. J., AUGSDÖRFER U. H., DODGSON N. A., SABIN M. A.: NURBS with extraordinary points: high-degree, non-uniform, rational subdivision schemes. *ACM Trans. Graphics* 28, 3 (July 2009), 46. 1

[CC78] CATMULL E., CLARK J.: Recursively generated B-spline surfaces on arbitrary topological meshes. *Computer-Aided Design* 10, 6 (Nov. 1978), 350–355. 1, 2

[CDS09] CASHMAN T. J., DODGSON N. A., SABIN M. A.: Selective knot insertion for symmetric, non-uniform refine and smooth B-spline subdivision. *Computer Aided Geometric Design* 26, 4 (May 2009), 472–479. 1

[DKT98] DE ROSE T., KASS M., TRUONG T.: Subdivision surfaces in character animation. In *SIGGRAPH '98* (July 1998), ACM Press, pp. 85–94. 1

[Far01] FARIN G.: *Curves and surfaces for CAGD: a practical guide*. Morgan Kaufmann, Oct. 2001. 1

[LS08] LOOP C., SCHAEFER S.: Approximating Catmull-Clark subdivision surfaces with bicubic patches. *ACM Trans. Graphics* 27, 1 (2008), 8. 1

[MFR*10] MÜLLER K., FÜNFIG C., REUSCHE L., HANSFORD D., FARIN G., HAGEN H.: Dinus: Double insertion, nonuniform, stationary subdivision surfaces. *ACM Trans. Graphics* 29, 3 (June 2010), 25. 1

[MRF06] MÜLLER K., REUSCHE L., FELLNER D.: Extended subdivision surfaces: Building a bridge between NURBS and Catmull-Clark surfaces. *ACM Trans. Graphics* 25, 2 (Apr. 2006), 268–292. 1

[SFL*08] SEDERBERG T. W., FINNIGAN G. T., LI X., LIN H., IPSON H.: Watertight trimmed NURBS. *ACM Trans. Graphics* 27, 3 (Aug. 2008), 79. 1

[Sta98] STAM J.: Exact evaluation of Catmull-Clark subdivision surfaces at arbitrary parameter values. In *SIGGRAPH '98* (July 1998), ACM Press, pp. 395–404. 1

[SZBN03] SEDERBERG T. W., ZHENG J., BAKENOV A., NASRI A.: T-splines and T-NURCCs. *ACM Trans. Graphics* 22, 3 (July 2003), 477–484. 1

[SZSS98] SEDERBERG T. W., ZHENG J., SEWELL D., SABIN M.: Non-uniform recursive subdivision surfaces. In *SIGGRAPH '98* (July 1998), ACM Press, pp. 387–394. 1

Further details

The work outlined in this extended abstract appears as a full paper in *Computer Aided Geometric Design*.
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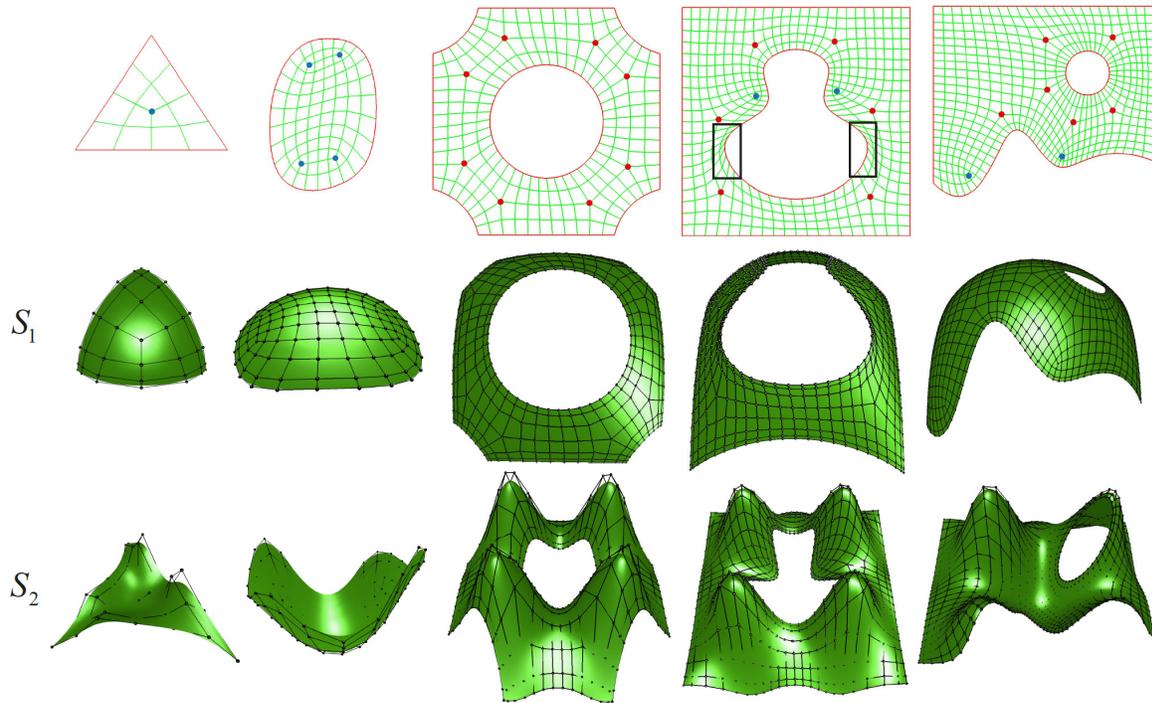


Figure 2: Conversion results. The first row shows different trimming loops and the corresponding decomposition in the trimmed domain region with 5-valent extraordinary vertices marked in red, 3-valent extraordinary vertices in blue. The second and third rows show the converted subdivision surfaces with two different NURBS surfaces (S_1 and S_2) as input surface. Note that they use the same decomposition in parameter space (the first row). S_2 has high curvature.

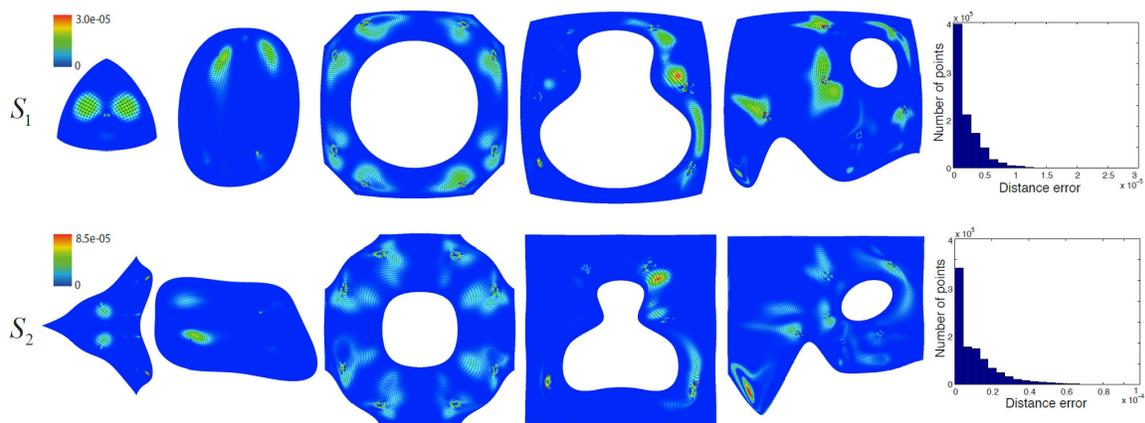


Figure 3: Distance error plots of conversion results in Figure 2. The error is scaled by the bounding box diagonal of the trimmed surface. The number of refinement steps is set to three to keep the approximation error below 10^{-4} . The histograms in the last column explicitly illustrate the error distribution of the examples in the fifth column.