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CAD Model Details via Curved Knot Lines and Truncated Powers ☆

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ABSTRACT

This paper describes a method for adding surface details to existing CAD models. Our approach is based on truncated powers, which allows us to align the added details with curved knot lines on the surface. Additionally, (truncated) powers give us precise control over the continuity of the perturbed surface across the (curved) knot lines. Our representation is compatible with current CAD/CAM practise and standards, and we showcase it on several examples.

1. Introduction

The automotive and aircraft industries are finding a problem in defining surfaces which are smooth enough when these surfaces have features which do not fit the rectangular structure of traditional NURBS surfaces. Individual NURBS patches are trimmed and (topologically) stitched together to form the so-called *B-rep*, i.e., the boundary representation of the desired model. Nevertheless, the individual B-rep faces locally maintain their tensor-product structure and adding detail not aligned with the existing knot line directions remains an issue.

We address this issue by the means of *truncated powers*, i.e., powers of functions defined implicitly and truncated to their non-negative region(s). This offers several advantages. First, it allows us to align the introduced detail along curved knot lines on the base surface (B-rep face). These knot lines are given by the zero-contours of the introduced implicit functions. Secondly, we have full control over the continuity of the perturbed surface across the curved knot lines, which is governed by the implicit functions and the powers they are raised to. Thirdly, as we are adding detail to an existing B-rep, the underlying representation can be based on (trimmed) NURBS, which is our focus, but also on other representations such as T-splines.

The rest of the paper is organised as follows. We start by reviewing related work in Section 2, which is followed by Section 3 introducing key notions. The main method is described in Section 4. Several examples are presented in Section 5 and the paper is concluded in Section 6.

☆ This paper has been recommended for acceptance by Jianmin Zhang.

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2. Related work

2.1. Bézier's System

In his book on his Unisurf system [1] (translated by Forrest into English [2, Section 4.2.5]), Bézier describes a technique for taking a bicubic patch spanning the entire side of a car, and trimming it to the relevant piece by applying a bicubic reparametrisation in which the required curved edges are isoparametric in the higher degree (up to bi-27) representation of the same 3D surface.

This meant that Unisurf had to be capable of holding surfaces of degree bi-27. This was misinterpreted by the CAD vendors who implemented this as 'you can define surfaces up to bi-27', which Bézier himself never did. Professional CAD systems now place no limitation on degree.

2.2. Hayes surfaces

In [3,4] the idea was introduced that while a rectangular grid of knot lines was retained, with a set of u-knots (values of u) and a set of v-knots, the individual knot values could vary with the other parameter.

Although the original reports deal only with bicubic splines, it is clear that the idea is completely applicable to any degree and to NURBS (Non-Uniform Rational B-Spline surfaces) [5] in their full generality.

This was presented at a conference where the proceedings were issued in the new hot medium of microfiche, which made the conference proceedings almost inaccessible [3]. There appear to be only 7 libraries worldwide (mostly in the UK) holding it. Luckily, the main ideas were recalled in [4].

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2.3. Trimmed NURBS

When the flow of shape in a surface varies significantly within a panel intended to be smooth for styling purposes, the current accepted approach is to use trimmed NURBS: a piecewise surface where the parametrisations do not match across piece boundaries. This links into downstream production processes such as machining of dies or additive manufacture and (with some difficulty) into analysis of the elastic properties of the panel and of the airflow around the panel. The format for passing the pieces to these downstream processes is usually IGES or STEP.

The automotive industry depends strongly on this approach. Unfortunately, achieving continuity of even position across the piece boundaries has been regarded as intractable. The auto industry has a standard often called Class-A surface [6] which specifies that pieces must meet with 0.1 mm, with a variation of surface normal direction within 0.1 degrees and with a further limit on discrepancy of curvature. This has proved satisfactory when panels are manufactured by pressing of metal sheets because the 'springback' of the metal as the die pressure is released allows the metal to forget these small discrepancies in the dies. However, if panels (typical internal ones) are to be made by additive manufacture, there is no springback, and the condition on normal direction allows creases to appear in the final shape. These are small, but not small enough. A tolerance of 0.1 degrees (≈ 0.002 radians) will cause a crest of more than 0.1 mm if the adjacent pieces are more than 5 mm wide. A width of 50 mm causes a crest of 1 mm, which may well be a lot more than the resolution of the additive manufacturing process and even comparable with the thickness of the part being made. If the error is a hollow rather than a crest, the problem is even worse, since polishing out a crease at the bottom of a hollow to give a smooth surface takes a much larger amount out of the material.

Although one solution would be to tighten the normal vector constraint on the standard, this would make the task of the software aids for making the pieces significantly harder (and for their operators).

There is a clear need for mathematics and software giving exact continuity of the necessary derivatives between the trimmed NURBS pieces of such composite surfaces.

2.4. T-splines and related methods

This was claimed by Sederberg et al. [7], but close examination of that paper shows that the mechanism for matching isoparametric lines to the piece boundaries is actually subdivision. Subdivision surfaces are not trimmed NURBS, and the transfer standards (IGES and STEP) and the downstream activities do not accept them. It is believed that because of this the Sederbergs' approach has been modified in their commercial software, replacing the shape around the necessary extraordinary vertices by higher degree patches which can be exported as individual NURBS pieces but this information is not publicly available.

There has been a lot of other work on constructions similar to T-splines, such as LR-splines [8] and THB-splines [9], but this has focused on keeping the density of basis functions sparse in regions which do not have high spatial frequency components, rather than on aligning the knots with features.

2.5. Simplex splines

Simplex splines [10] are optimal in terms of the trade-off between degree and continuity. The problems for application in the styling of objects to be manufactured (as against the graphics applications) are twofold: First, the actual surface consists of many more tiny pieces which would each need to be exported via IGES independently. This is because each simplex spline basis function consists of a large number of individual polynomial fragments, and these have to be very carefully designed to avoid very narrow ones (slivers). Second, the support of each basis function is a polygon, and so curving the boundary even of a collection of such basis functions is not possible.

2.6. Watertight Boolean operations

This recent method [11] reparametrises NURBS patches affected by trimming curves so that these become iso-lines of the patches. This then allows for a watertight connection of the patches as the B-rep edges can, after this reparametrisation and slight perturbation, be exactly interpolated. In some cases, extraordinary points need to be introduced, for now largely manually, to deal with some of the arising configurations.

2.7. ABC patches

The first serious attempt started in the mid 2010s, the first publication to describe it being [12]. This was followed up by [13] which claimed up to G^2 continuity, but the claim in [12] that the mathematics supports general levels of derivative continuity is totally plausible. (This may be a question of 'sweet spot' for implementation mentioned in Section 4.4.)

The idea is based on transfinite interpolation between the boundaries of the known pieces implemented as a correction to base surfaces. Transfinite surfaces between NURBS boundary curves are themselves representable as NURBS and so the necessary compatibility with the downstream activities is maintained.

Current commercial CAD techniques do not provide exact continuity, and so the process of styling a shape to meet aesthetic requirements within industry-accepted tolerances on continuity is non-trivial. Our method introduces details to existing B-rep faces, which means that it can build on top of commercial CAD system representations as well as all the base-surface methods reviewed here, as well as various finite filling constructions [14] and even subdivision surfaces [15] (although the latter are currently not directly CAD-compatible).

3. Preliminaries

Although the original definition of a *spline* is a piecewise polynomial map $\mathbb{R} \mapsto \mathbb{R}$, we unashamedly generalise this to domains and ranges of higher dimension. Where necessary this will be made explicit by using the terms *spline curve* and *spline surface*.

A spline curve is a map $\mathbb{R} \mapsto \mathbb{R}^2$ (a 2D spline curve) or $\mathbb{R} \mapsto \mathbb{R}^3$ (a 3D spline curve). A spline surface is a map $\mathbb{R}^2 \mapsto \mathbb{R}^3$. In these maps the abscissa/argument values are called *parameters*.

A *knot* is an abscissa value where polynomial pieces join, where there is at least a potential discontinuity of some derivative. In the curve case it is just a parameter value. In the surface case, the tensor product structure forces any knots to be isoparametric lines, and it is this limitation which is removed here. We permit a knot in a surface to be a curve in parameter space. In the terminology of CAD systems such a curve is called a *p-curve*.

In CAD, a solid object is represented by its boundary, the so-called B-rep. This is held in data supporting computations in the form of a Whitney stratification of space into *cells*, which are manifold open-regular pointsets of different dimension together covering the entire boundary of the object. The full stratification also includes solid cells for the interior. Cells can be *vertices*, *edges* or *faces*. The adjacency relationships of these cells are commonly called the *topology* of the representation.

Each cell also has an *embedding*, which is a pointset in which the vertex, edge or face lies. A vertex is embedded in a *point*, an edge in a *curve* and a face in a *surface*. These are collectively called the *geometry* of the representation. A collection of surfaces sharing the same parameter space is called a *carpet*.

We treat as axiomatic certain lemmas from Algebraic Geometry. A rational polynomial curve (not piecewise) in 2D is a pointset which can be represented in its parametric form by a map $\mathbb{R} \mapsto \mathbb{R}^2$. Each such curve also has a representation of the *implicit* form $f(.) = 0$ where f takes positive values on one side of the curve and negative values on the other. It is therefore usefully understood as the map $\mathbb{R}^2 \mapsto \mathbb{R}$. The implicit form has the same degree as the parametric one. Not every implicit curve has a parametric equivalent.

4. General theory

4.1. Overall Carpet structure

We propose that the definition of the entire carpet be the sum of: (1) a *base surface* $\mathbf{B}(u, v)$ which can itself be a standard NURBS (or another spline representation over a single parametric domain), and (2) a collection of *perturbations* (or details) which tend to correspond to desired features of the final composite surface. These are called 'TWEAKS' in the definition file; see Fig. 4. The definition is completed by a collection of overall trimming curves which bound the entire carpet. These are called 'TRIMS' in the definition file; see Appendix A.

The entire surface is a map from a single parameter space, and so we do not need to be concerned about G continuity versus C continuity: there is exact C continuity of the degrees demanded.

Each perturbation is defined in terms of its *outline* which is the frontier between itself and the rest of the surface, and a displacement *coefficient*, which is a constant displacement [16] indicating how much the feature is moved from the corresponding point of the underlying surface. This displacement does not need to be aligned with the normal direction of the base surface.

Perturbations can overlap, without concern, except that if the outlines make tangential contact this will cause 'slivers' which may cause difficulties in the downstream operations when there is floating point rounding error in the transfer formats. Tangential contact is better avoided.

4.2. Outlines

We use as outlines complete closed polynomial curves which have both an implicit representation ($\mathbb{R}^2 \mapsto \mathbb{R}$) and a piecewise rational parametric representation. By convention the non-zero part of the perturbation lies in the positive region of the implicit form. Because each of the parametric pieces becomes a p-curve separating two pieces of the total surface, its direction is not constrained.

4.3. Interior form of a perturbation

The displacement of each interior point of a perturbation is given by the product of the coefficient with a function of parametric position given by a *truncated power* of the function of the implicit form.

A truncated power $f_+^n(\mathbf{p})$ of a function $f(\mathbf{p})$ at point $\mathbf{p} = (u, v)$ in parameter space is defined to be

$$f_+^n(\mathbf{p}) = \begin{cases} f(\mathbf{p})^n & \text{if } f(\mathbf{p}) > 0 \\ 0 & \text{otherwise} \end{cases}$$

This function f_+^n has a discontinuity of nth derivative across $f(\mathbf{p}) = 0$, which is exactly what we are trying to construct. If $f(\mathbf{p})$ is itself a polynomial of degree d, then $f_+^n(\mathbf{p})$ is a polynomial of degree nd in the region where $f(\mathbf{p}) > 0$.

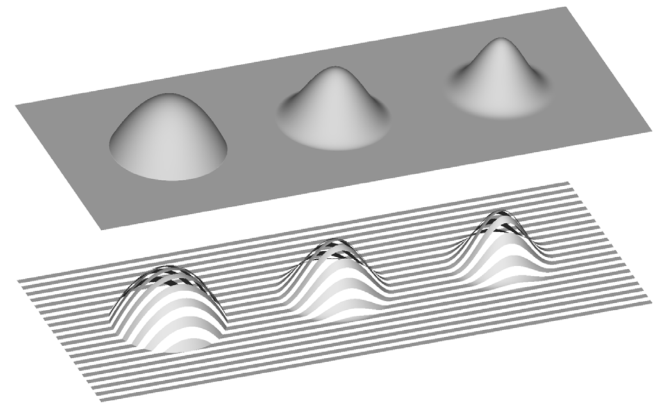

Fig. 1. An illustration of the sweet spot. Using the same circle equation $f = 1 - u^2 - v^2$ raised to different powers (from left to right: 1, 2, and 3) yields perturbations with different continuities (from left to right: C^0, C^1, and C^2). The displacement was in all three cases set perpendicular to the base plane: $\mathbf{C} = (0, 0, 1)$. The top row shows the shaded surface and the bottom row shows the same surface using slices.

Admissible functions f include linear ones, quadratic functions (discussed in detail in Section 4.4), and higher degree ones which admit a (piece-wise) rational parametrisation of their zero contour (are of genus zero).

Each exported piece has an equation of the form

$$\mathbf{P}(u, v) = \mathbf{B}(u, v) + \Sigma_i \mathbf{C}_i f_+^{n_i}(u, v), \tag{1}$$

where $\mathbf{B}(u, v)$ is the base surface, i counts through the tweaks active in that piece, $\mathbf{C}_i$ is the ith displacement-valued coefficient also known as a control vector [16], and n_i is the degree of continuity of that tweak.

Thus each intersection of interiors has as its equation the sum of the base surface (a bi-polynomial/bi-rational function) and a number of perturbations, each of which is a bi-polynomial, so that its final shape is also bi-polynomial or bi-rational, which can be exported to IGES or STEP. The explicit parametric forms of the boundaries of the perturbations form the trimming curves of the representation.

There is no inherent limit to the degrees involved, but high degrees of d or n imply that the final surface may be of very high degree. For CAD/CAM purposes this is not very desirable, and so limited values give a 'sweet spot' for implementation.

4.4. Sweet spot

It appears that the use of functions $f(\mathbf{p})$ limited to quadratic outlines (thus avoiding all problems with multiple points except for degenerate cases) and truncated powers limited to degree 3 (thus supporting surfaces which have continuity of derivative up to C^2) gives a useful operational capability; see Fig. 1. If the base surface is bicubic this gives surfaces of total degree 6, which can be exported as bi-6 NURBS and trim curves (evaluated by exporting rational quadratic parametric p-curves on a degree 6 surface) of rational degree 12. Both degrees are well within the capability of both of the transfer standards (IGES and STEP) and professional CAD systems.

These limitations are not inherent in the idea itself and can be stretched. The penalty for increasing the degree of the outlines is that more care must be taken with self-intersection points, and for increasing the degree of continuity is that the final degree of the surface pieces and trim curves will be higher. These might be acceptable in a professional implementation.

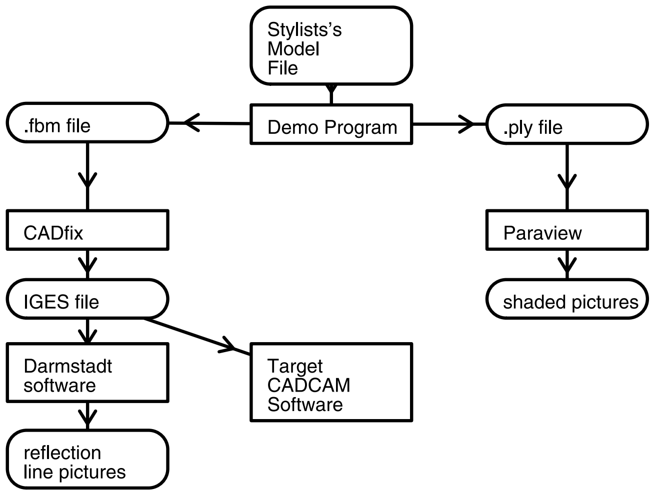

Fig. 2. This program was used to make the examples, using the CADfix product [17] (via the .fbm file) to interface through to the IGES format and for making the illustrations, and the Paraview product (via .ply files) to provide a good way of viewing the shapes created. The reflection lines were created from the IGES file by Darmstadt Technical University using MATLAB-based software.

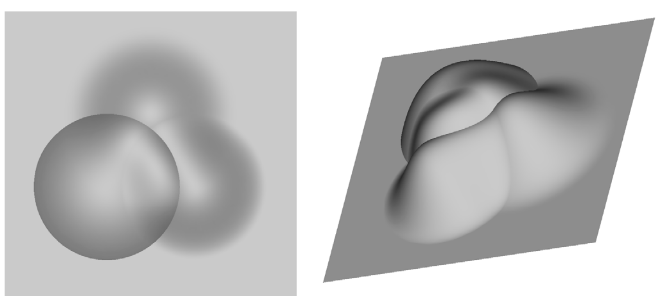

Fig. 3. These figures illustrate how features with different degrees of derivative continuity can be combined. The model has a flat base surface with three overlapping features each being inside a circular outline. On the left appears a plan view where the three different continuity levels (C^0, C^1 and C^2) are visible in the greyscale: The bottom left feature joins C^0, the bottom right C^1, and the top one C^2. On the right is a view of the resulting surface from a skew angle and again the different continuity degrees are visible, crossing the other features where they overlap.

4.5. Status

Much of this theory has been implemented in a Delphi3 program for Windows. It is limited to the sweet spot of degrees, but is further limited to having only elliptical (and straight line) parametric trim curves, not hyperbolae. The dataflow diagram appears in Fig. 2.

The IGES format does not support the parametric trim lines explicitly. They are recovered if necessary by the downstream software by projection on to the relevant surfaces. Since the space-curves issued will lie exactly on these surfaces, this process can be expected to be robust.

5. Examples

The simple example shown in Fig. 3 uses a flat base surface and has three perturbations. Each perturbation has a circular outline, but they vary in the continuity degree across those outlines. The input data file is shown in Fig. 4.

We created a fictional shape similar to the surface covering the front wheel of a car, with a small feature for an auxiliary light at the front; see Fig. 5. This region is typically a hard one

```
BASE DUMMY // A flat base in z = 0 with (x, y) ∈ [−1, 1]^2
TWEAK ELLIPSE 2 // Each TWEAK defines a perturbation with
elliptical outline.  The numerical argument is the degree
of derivative continuity required
0.5 0.65 // centre in parameter space
0.5 0.95 //two conjugate points on outline
0.8 0.65
0 0 .2 // the displacement vector to be applied at the
centre of the perturbation
TWEAK ELLIPSE 1 // The TWEAKS can be entered in any
sequence
0.65 0.4
0.9 0.4
0.65 0.65
0 0.0 0.2
TWEAK ELLIPSE 0
0.35 0.4
0.6 0.4
0.35 0.65
0.0 0.0 0.2
```

Fig. 4. Input data file for Fig. 3.

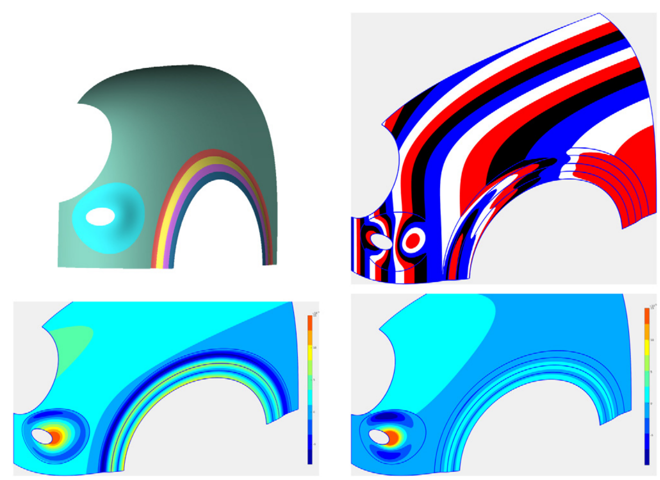

Fig. 5. Figures created from the IGES file, showing the curvature behaviour. Top left shows the regions within the overall carpet, top right shows the reflection lines. Although in some places these do not appear C^1, zooming in indicates that the complete surface is indeed C^2. Bottom left shows the distribution of mean curvature and bottom right that of Gaussian curvature. In the region where there are rapid changes of direction of the reflection lines, the curvature, though continuous, is also changing rapidly. The full input file can be found in Appendix A, Fig. 7.

to represent, needing several trimmed NURBS patches with only approximate continuity. Fig. 6, left, shows the knot curves in the parameter plane. The right image depicts a cross section through the rim around the wheel hole. It shows the knots and a very similar cubic B-spline. The similarity is not exact because the cross-section has sextic pieces not cubic, but the difference tends to zero as the distance between knots as a fraction of the diameter of the ellipses becomes small.

6. Conclusion

A method has been presented for designing single curved surfaces with features flowing in different directions, which can be transferred via IGES or STEP to the downstream analysis and manufacture operations. The method uses two key ideas, of curved knots and truncated powers to achieve this and has been demonstrated by examples. In principle arbitrary degrees of continuity can be achieved, at the cost of high degree, but C^2 involves only

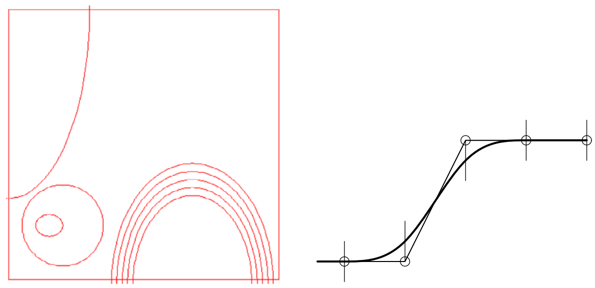

Fig. 6. Left: The layout of curved knots around the wheel cutout, showing how 'parallel' curved knots give an effect similar to familiar B-spline control. Right: A cross section through the rim.

degree bi-6 pieces which is well within practical limits. Several potential extensions are discussed in Appendix B.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. More complex input file example

Fig. 7 lists the full input file for the model shown in Fig. 5.

```
BASE BEZIER
0 300 0
0 0 0
500 0 0
1000 0 0
0 300 300
0 0 300
500 0 300
1000 0 300
0 300 300
0 0 600
500 0 600
1000 0 600
0 300 300
0 300 600
500 300 600
1000 300 600
TWEAK ELLIPSE 2 // This is one of the tweaks giving the shape of the rim of the hole for the
wheel.
0.68 -0.02
0.98 -0.02
0.68 0.43
0 -1000 0
TWEAK ELLIPSE 2
0.68 -0.02
0.96 -0.02
0.68 0.40
0 2000 0
TWEAK ELLIPSE 2
0.68 -0.02
0.94 -0.02
0.68 0.37
0.0 -1500.0 0.0
TWEAK ELLIPSE 2
0.68 -0.02
0.92 -0.02
0.68 0.34
0.0 1000.0 0.0
TRIM ELLIPSE INSIDE // This is the headlight hole, which trims out the really ugly part of the
base surface (which could be described much more elegantly in terms of a Bézier triangle.
-0.001 1.001 // centre
0.3 1.001 // two conjugate points on the ellipse.
-0.001 0.3
TRIM ELLIPSE INSIDE
0.68 -0.02
0.90 -0.02
0.68 0.31
TWEAK ELLIPSE 2
0.2 0.2
0.35 0.2
0.2 0.35
-20 -25 0.0
TRIM ELLIPSE INSIDE
0.15 0.2
0.2 0.2
0.15 0.24
```

Fig. 7. A more complex input file example; see Fig. 5.

Appendix B. Desirable enhancements

The following are possible enhancements which could be made from the current proof of concept implementation to a fuller sweet-spot version. Increasing the degrees to full generality would require significant design effort before implementation.

Degree 6 base surfaces

Allow the use of a degree 6 Bézier triangle as an alternative to a bicubic for the base surface. This requires a little extra coding, as a bicubic base has to be converted to a degree 6 triangle as soon as a perturbation is added.

Hyperbolae as knots

Add hyperbolae to the possible p-curves. This opens up the option of requiring just one half of the hyperbola to be used, which can be achieved by using an *auxiliary selector*, as described next.

Auxiliary conditions

If we require only part of $f(\mathbf{p}) = 0$ to act as a knot, then changing the definition of f_+^n to

$$f_+^n(\mathbf{p}) = \begin{cases} f(\mathbf{p})^n & \text{if } f(\mathbf{p}) > 0 \text{ and } g(\mathbf{p}) > 0 \\ 0 & \text{otherwise} \end{cases}$$

allows only that part of $f = 0$ which lies within $g > 0$ to form the actual knot. The shape within $f(\mathbf{p}), g(\mathbf{p}) > 0$ is unchanged. Note that the detail of the shape there does not depend on the exact shape of g, which can be chosen to separate the desired part of the perturbation from an undesired part. This idea will be relevant if higher degrees of outline are used, when the parametric form of the outline will in general have either self-intersection points or (multiple) loops. A particularly useful case appears in the sweet-spot, when the hyperbola consists of two straight lines, but only one quadrant is required for the perturbation, giving it a sharp convex corner. Another example might be to separate the tear-drop shape of a Tschirnhausen cubic from the part beyond the self-intersection point. For such cases (of degree lower than 4), g can simply be linear.

'Parallel' knots

If we have a number of knots which use scalar offsets of a common function, the knots together behave very similarly to standard isoparametric knots defined over a reparametrisation of the domain. Thus a lot of standard tensor-product B-spline theory can be carried over and exploited to make the specification of the desired shape simpler. Actually doing the reparametrisation on the active side of the outermost knot is probably not necessary, and is not explored further here. The theory of plane Cremona transformations [18] may be relevant, but is also beyond the scope of this paper.

Non-constant coefficients

We used displacement-valued coefficients, so that the centre of a tweak could be moved in any direction, not just normal to the base surface. It would be possible to use coefficients themselves containing a polynomial function of u and v, thus increasing the range of shapes definable. We did not include this because it would complicate the stylist's task, but it could be useful in defining nested functions for analysis or optimisation.

Version with graphical input of the stylist's data

This would require a better local rendering (with adequate speed) instead of the use of the .ply file, together with the use of graphical input to specify the points defining each tweak.

Automatic generation of data from a clay model scan

An ideal part of the work-flow would be software which created the input data from a dense triangulation such as might be created by laser-scanning a clay model. This is highly non-linear, but might be addressed by the kind of technology used by Barrowclough et al. [19].

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