Differential Attribute Profiles in Remote Sensing

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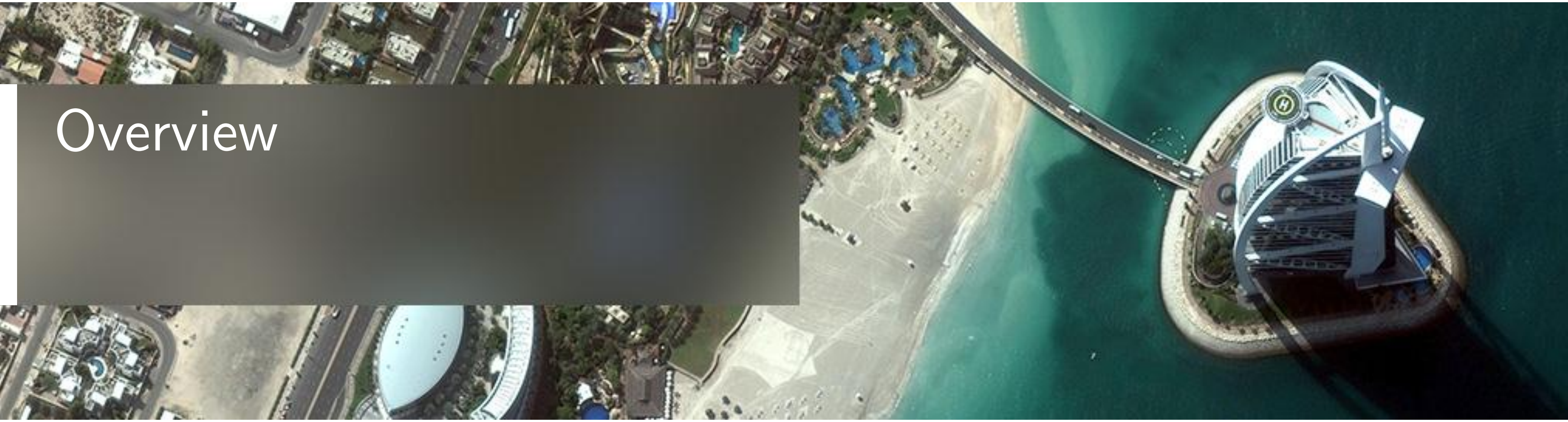
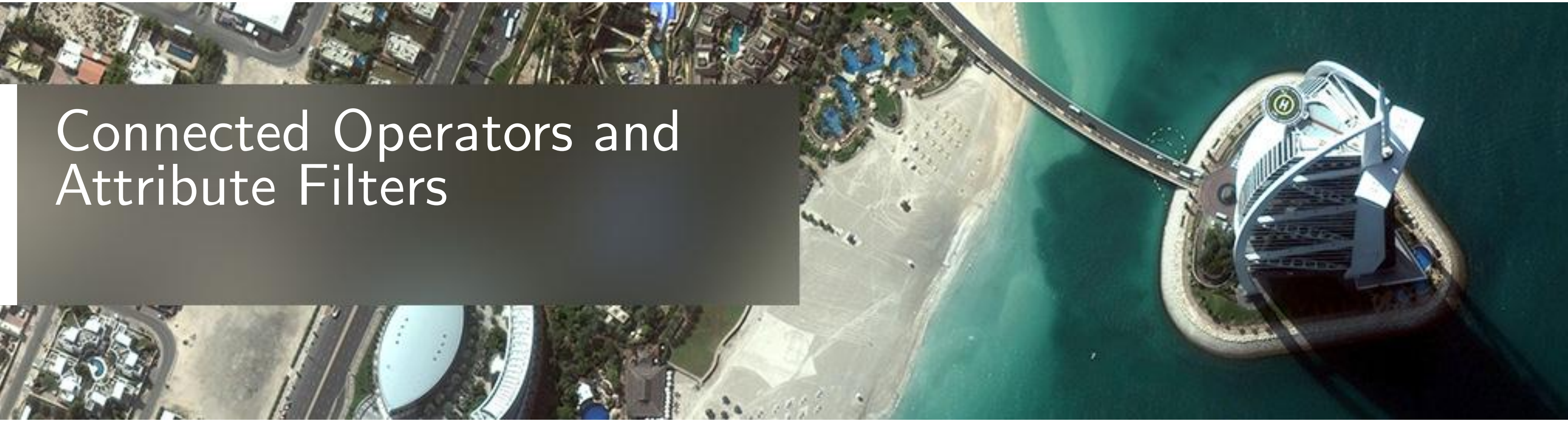


Overview

- Connected Operators and Attribute Filters
- Granulometries and Pattern Spectra
- Differential Attribute Profiles
- The CSL Model
- Applications in Remote Sensing
- Conclusions



Connected Operators and Attribute Filters

Notations

- **Binary set:** X is defined as a subset of the image definition domain E.
- **Powerset:** $\mathcal{P}(E)$ is the set of all subsets of E.
- Operators on binary images are denoted with capital Greek letters.
- An operator followed by a lowercase subscript $x \in E$ is a connected operator on a binary set marked by a point x.

Connected Attribute Filters

A filter in mathematical morphology is defined as an operator:

$$\Psi : \mathcal{P}(E) \rightarrow \mathcal{P}(E) \tag{1}$$

that is increasing and idempotent.

For any two binary sets $X, Y \subseteq E$, the two properties mean:

- **Increasing:** $X \subseteq Y \Rightarrow \Psi(X) \subseteq \Psi(Y)$
- **Idempotent:** $\Psi(\Psi(X)) = \Psi(X)$

Connected Attribute Filters

Connected filters define a more explicit class of operators that process connected image regions $C \subseteq X$ known as connected components of an image X, instead of individual points.

- They are **edge preserving** operators; they can either remove a connected component in its entirety or leave it intact.
- This prevents **shape/size distortion** often associated with other types of operators.
- If this decision is based on some component attribute the operator is referred to as an **attribute filter**.

Connected Attribute Filters

Increasing attribute filters are based on **connectivity openings** or **closings**.

Non-increasing attribute filters are based on **connectivity thinnings** or **thickennings**.

- A connectivity opening $\Gamma_x(X)$ given a point $x \in X$, returns the connected component containing x or $\emptyset$ otherwise.
- For all $X \subseteq E$, and $x, y \in E$, $\Gamma_x(X)$ and $\Gamma_y(X)$ are either equal or disjoint.

Connected Attribute Filters

- Attribute filters retain only those connected components that satisfy some pre-specified criteria.
- A criterion enumerates the logical output of a condition typically given as $\Lambda(C) \geq \lambda$, in which $\Lambda(C)$ is some real-value attribute of C, and λ an attribute threshold.
- The criterion, by analogy to an operator, is increasing if for any two ordered inputs, the respective outputs retain the same order.

Criteria are put in place by means of a trivial opening $\Gamma_{(\Lambda,\lambda)}$, for which $\Gamma_{(\Lambda,\lambda)}(\emptyset) = \emptyset$, and for any other $C \subseteq E$:

$$\Gamma_{(\Lambda,\lambda)}(C) = \begin{cases} C & \text{if } \Lambda(C) \geq \lambda, \quad (2a) \\ \emptyset & \text{otherwise.} \quad (2b) \end{cases}$$

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Definition 1. *The binary attribute opening* Γ_λ^Λ *of a set* X*, subject to an increasing criterion, is given by:*

$$\Gamma_\lambda^\Lambda(X) = \bigcup_{x \in X} \Gamma_{(\Lambda,\lambda)}(\Gamma_x(X)). \tag{3}$$

Attributes can be either increasing such as the component area, the size of structuring element fitted into the component, etc., or non-increasing such as compactness, contour smoothness, moments of inertia, etc.

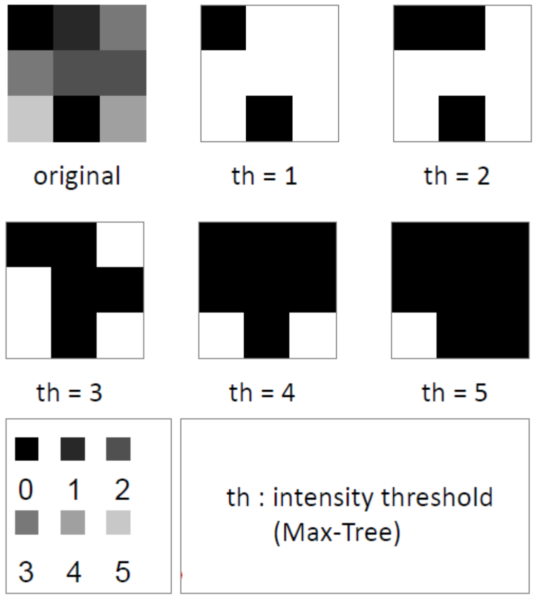
Connected Attribute Filters

Attribute filters extend to gray-scale through threshold superposition.

Let $f : E \rightarrow \mathbb{Z}$ be a gray-scale image, decomposed to a set of threshold sets, each given by:

$$T_h(f) = \{x \in E \mid f(x) \geq h\}, \quad (4)$$

in which the level $h \in \{h_{\min}, ..., h_{\max}\}$.



Connected Attribute Filters

For any two levels $h < h' \Rightarrow T_h(f) \supseteq T_{h'}(f)$ and moreover for any increasing criterion:

Definition 2. *The response of an attribute opening on a mapping $f : E \to \mathbb{Z}$, at a point $x \in E$ is given by:*

$$(\gamma_\lambda^\Lambda(f))(x) = \vee \left\{ h \mid x \in \Gamma_{(\Lambda,\lambda)}(\Gamma_x(T_h(f))) \right\}. \tag{5}$$

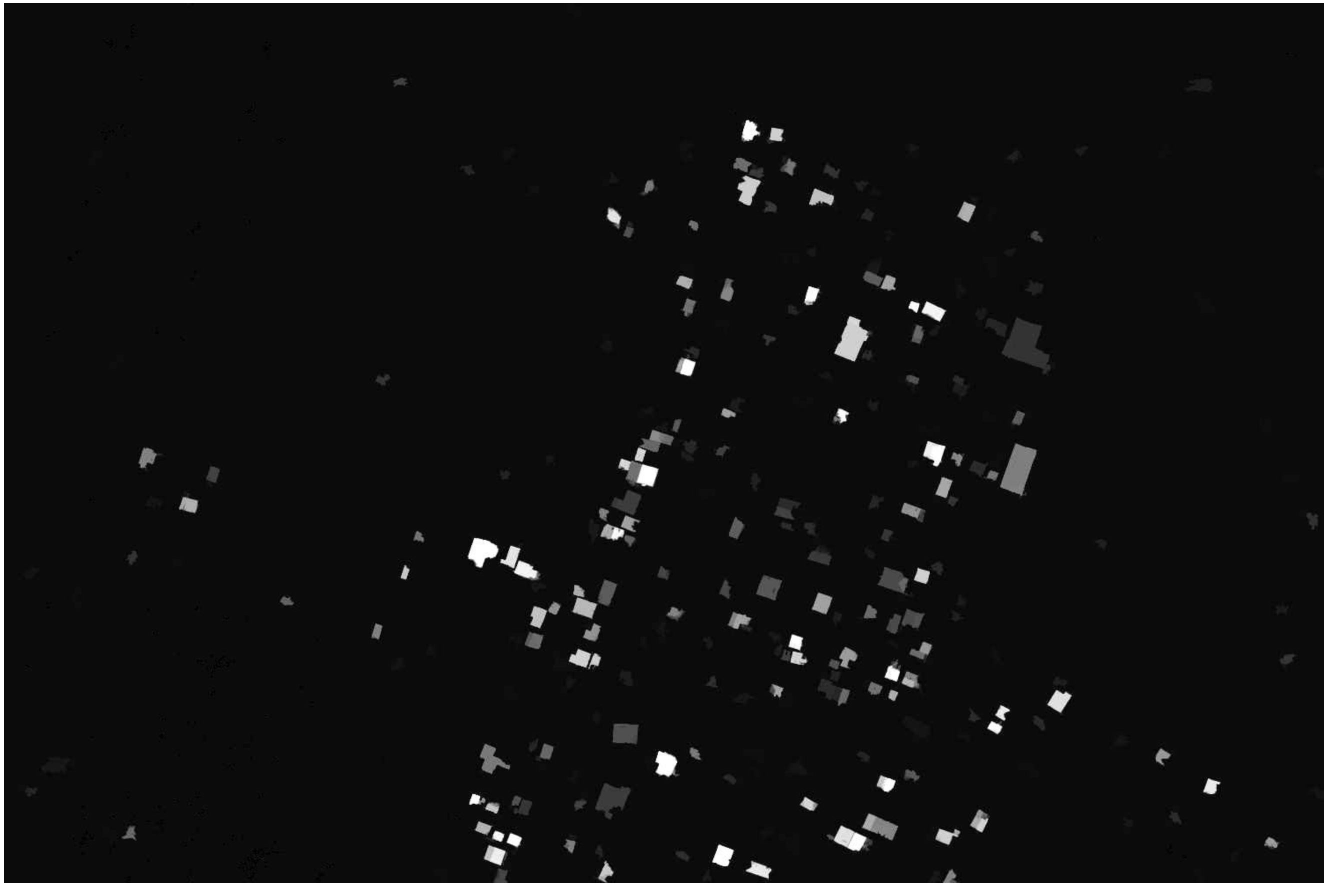
In brief, the attribute opening of a gray-scale image assigns to each point of the original image the highest level-threshold at which it still belongs to a connected foreground component.

Gray-scale attribute closings ϕ_λ^Λ are defined analogously.

Example of Attribute Filtering

Aerial photography of rural settlements in Guatemala (courtesy of Government of Guatemala, World Bank).

Example of Attribute Filtering



Output image: a sequence of filters based on size, compactness and contour smoothness criteria (attribute filters).

Granulometries and Pattern Spectra

Granulometries and Pattern Spectra

- In the study of attribute filters in multi-scale applications, a useful construct was introduced known as **granulometry**.
- A granulometry is a family of filter instances computed from a vector of ordered attribute thresholds $\vec{\lambda}$.
- Granulometries offer an easy way to study the distribution of image detail with respect to the concerned attribute.

Let $\vec{\lambda} = \{\lambda_0, \lambda_1, ..., \lambda_{max}\}$, be a set of totally ordered thresholds for some increasing attribute.

Granulometries and Pattern Spectra

- Formally, a granulometry of a binary image X can be defined as a decreasing family of attribute openings $\{\Gamma^{\lambda} \mid \lambda \in \vec{\lambda}\}$ for which:

$$\forall \lambda, \lambda' \in \vec{\lambda} \Rightarrow \Gamma^{\lambda}(\Gamma^{\lambda'}(X)) = \Gamma^{\lambda'}(\Gamma^{\lambda}(X)) = \Gamma^{\max(\lambda,\lambda')}(X). \qquad (6)$$

- Granulometries for non-increasing operators have been investigated by Urbach et al. The order in that case is preserved by utilizing appropriate filtering rules.

Granulometries and Pattern Spectra

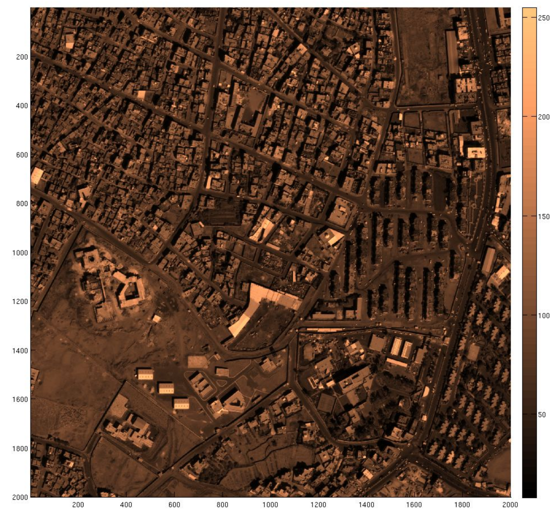
- The distribution of image detail that is often given by the sum of pixels, with respect to one or more attributes is a histogram that is referred to as the granulometric curve or *pattern spectrum*.
- An attribute class or histogram bin to which a point $x \in X$ contributes, is the smallest value of λ for which $x \notin \Gamma^{\lambda}(X)$.
- The pattern spectrum by $\mathtt{PS}_{\Gamma^{\lambda}}(X)$ applying an attribute-specific granulometry $\{\Gamma^{\lambda}\}$ to a binary image X is defined as:

$$(\mathtt{PS}_{\Gamma^{\lambda}}(X))(u) = -\left.\frac{\mathrm{d}\xi(\Gamma^{\lambda}(X))}{\mathrm{d}\lambda}\right|_{\lambda=u}. \tag{7}$$

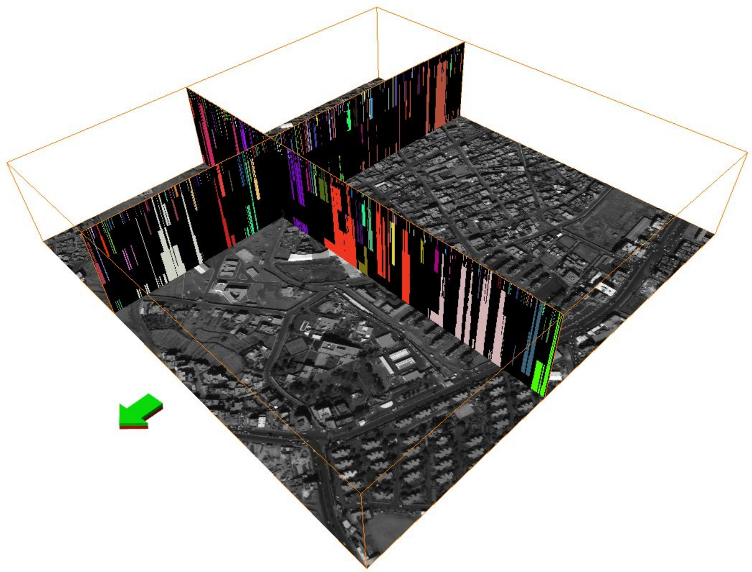
- The term ξ is the Lebesgue measure, which for $n = 2$ is the set area.

Differential Attribute Profiles

Differential Attribute Profiles



A residential area in Sana'a, Yemen. Panchromatic Quickbird image with 0.6 spatial resolution; courtesy of DigitalGlobe, Inc.



Two sets of DAP vectors; set of positive vectors faces the direction of the arrow; set of negative vectors face the direction perpendicular to the arrow.

The **Differential Attribute Profile** or **DAP** of a point $x \in E$ is a **spatial signature** consisting of two response vectors; the *positive* and *negative vector*.

Differential Attribute Profiles

Consider a pair of attribute filters $(\gamma_{\lambda}^{A}, \phi_{\lambda}^{A})$ that define an adjunction and an attribute threshold vector $\vec{\lambda} = [\lambda_i]$ with $i \in [0, 1, 2, ..., I - 1]$ and $\lambda_0 = 0$.

Operating each of the two attribute filters for the thresholds given in $\vec{\lambda}$ produces two granulometries;

- one in which bright information are progressively reduced (based on the opening/thinning),
- and a second in which dark information are progressively reduced (based on the closing/thickenning).

Differential Attribute Profiles

For each filter, let Δ^{Π} be a differential profile that given a point $x \in E$ is defined as follows:

$$\Delta^{\Pi}(\gamma_{\lambda}^{A}(f))(x) = \Big((\gamma_{\lambda_{i-1}}^{A}(f) - \gamma_{\lambda_{i}}^{A}(f))(x) \mid \lambda_i > \lambda_{i-1}, \forall\, i \in [1, ..., I-1] \Big) \quad (8)$$

for the filter γ_{λ}^{A}, and

$$\Delta^{\Pi}(\phi_{\lambda}^{A}(f))(x) = \Big(-(\phi_{\lambda_{i-1}}^{A}(f) - \phi_{\lambda_{i}}^{A}(f))(x) \mid \lambda_i > \lambda_{i-1}, \forall\, i \in [1, ..., I-1] \Big) \quad (9)$$

for ϕ_{λ}^{A}.

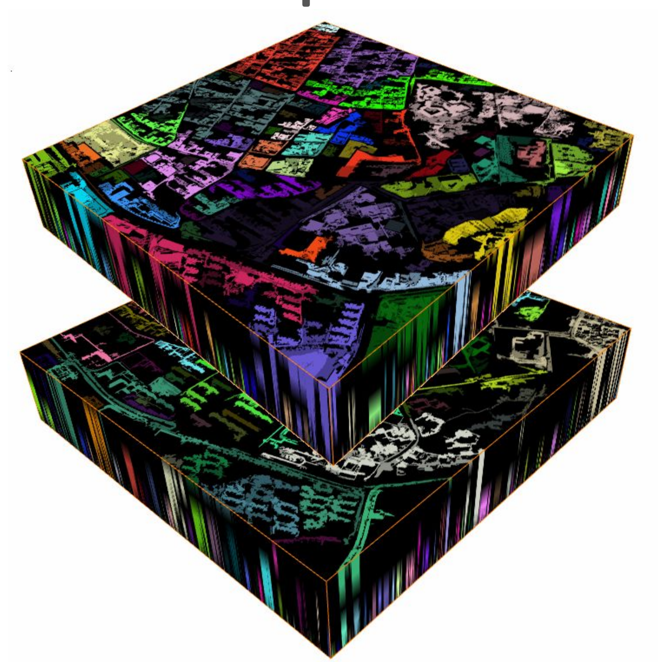
Differential Attribute Profiles

- Each profile is a response vector associated to a point $x \in E$.
- $\Delta^{\Pi}(\gamma_\lambda^A)$ is called the positive response vector.
- $\Delta^{\Pi}(\phi_\lambda^A)$ is called the negative response vector.
- The concatenation of the two, denoted with $\sqcup$, is a $2 \times (I-1)$ long vector, called the differential attribute profile of x:

$$\mathsf{DAP}(x) = (\Delta^{\Pi}(\gamma_\lambda^A(f)) \sqcup \Delta^{\Pi}(\phi_\lambda^A(f)))(x). \tag{10}$$

Differential Attribute Profiles

- The set of DAPs computed from the full extent of image definition domain is called the **DAP vector field**.
- An instance i of a DAP vector field is a difference image with respect to the given operator and is called a **DAP plane**.



The DAP vector field computed from the image of Sana'a.

Differential Attribute Profiles

Computing DAPs efficiently: Image Decomposition to Attribute zones

- Attribute filters acting on a gray-scale image produce a **dichotomy**, i.e. they separate the image contents in two sets of components; those that satisfy the attribute criterion and those that fail it.
- Extending this for multiple attribute thresholds gives raise to the concept of **polychotomy** and results in a image decomposition to *attribute zones*.
- Each zone is a grouping of peak components with attribute values being within some pre-specified range.

Differential Attribute Profiles

Let Γ_λ^A and Φ_λ^A be the binary counterparts of γ_λ^A and ϕ_λ^A respectively, and consider a threshold decomposition of f.

Each threshold set $T_h(f)$, with $h \in \{h_{min}, ..., h_{max}\}$, contains $k \in K_h^f$ peak components P_k^h, and for each pixel $x \in E$ the characteristic function χ of $T_h(f)$ is given by:

$$(\chi(T_h(f)))(x) = \begin{cases} 1 & \text{if } x \in T_h(f), \\ 0 & \text{otherwise}, \end{cases} \tag{11}$$

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Differential Attribute Profiles

Definition 3. *The* attribute zone $\zeta^A_{\lambda_a,\lambda_b}$ *of a mapping* $f : E \rightarrow \mathbb{Z}$*, bounded by two attribute thresholds* λ_a *and* λ_b *such that* $\lambda_a < \lambda_b$*, is given by:*

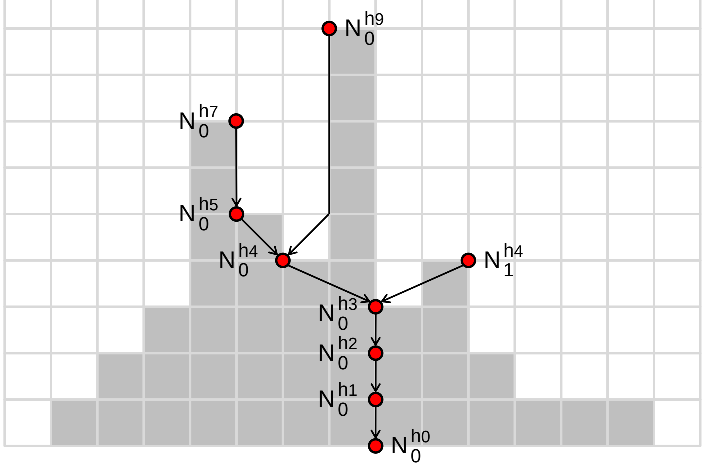
$$\zeta^A_{\lambda_a,\lambda_b}(f) = \sum_{h=h_{\min}+1}^{h_{\max}} \Big(\sum_{k \in K^f_h} \chi\Big(\Gamma^A_{\lambda_a}(P^h_k) \setminus \Gamma^A_{\lambda_b}(P^h_k)\Big)\Big). \tag{12}$$

The intensity of any point x of the image domain, that marks a component in a zone $\zeta^A_{\lambda_a,\lambda_b}$ can be obtained by initializing it to zero and updating it by adding the value of 1 for each level at which, $x \in P^h_k$ is non-zero in the difference between the two attribute filters $\Gamma^A_{\lambda_a}$, $\Gamma^A_{\lambda_b}$.

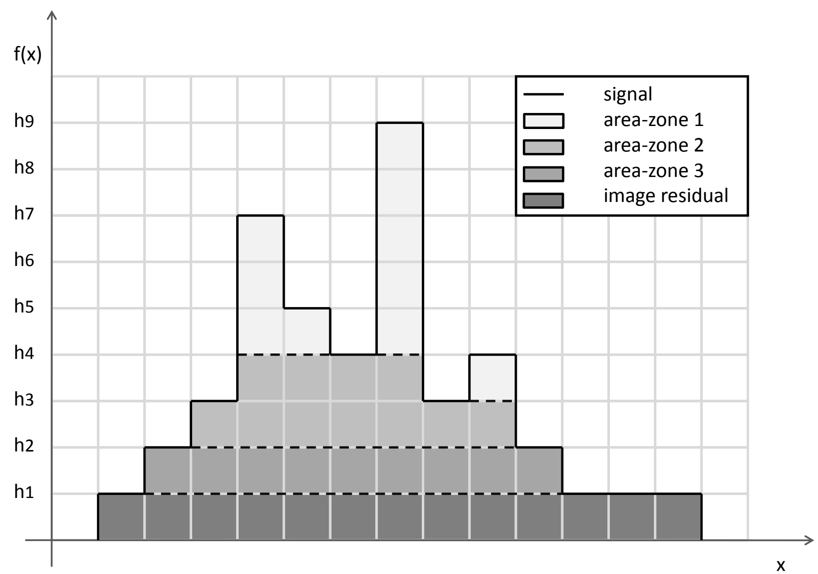
The same decomposition is defined for the operator Φ^A_λ.

Differential Attribute Profiles

The attribute zone operator is a connected operator and it generates an attribute-based decomposition, i.e. any two zones do not overlap and the decomposition is unique.



Input signal with its Max-Tree.



Signal decomposed to area zones.

Example of area zone decomposition with $\vec{\lambda} = [0, 4, 8, 12]$.

Differential Attribute Profiles

- The attribute zone operator is connected because it generates a coarser partition compared to the partition of flat zones of the input image.
- The attribute-based decomposition properties can be summarized as follows:

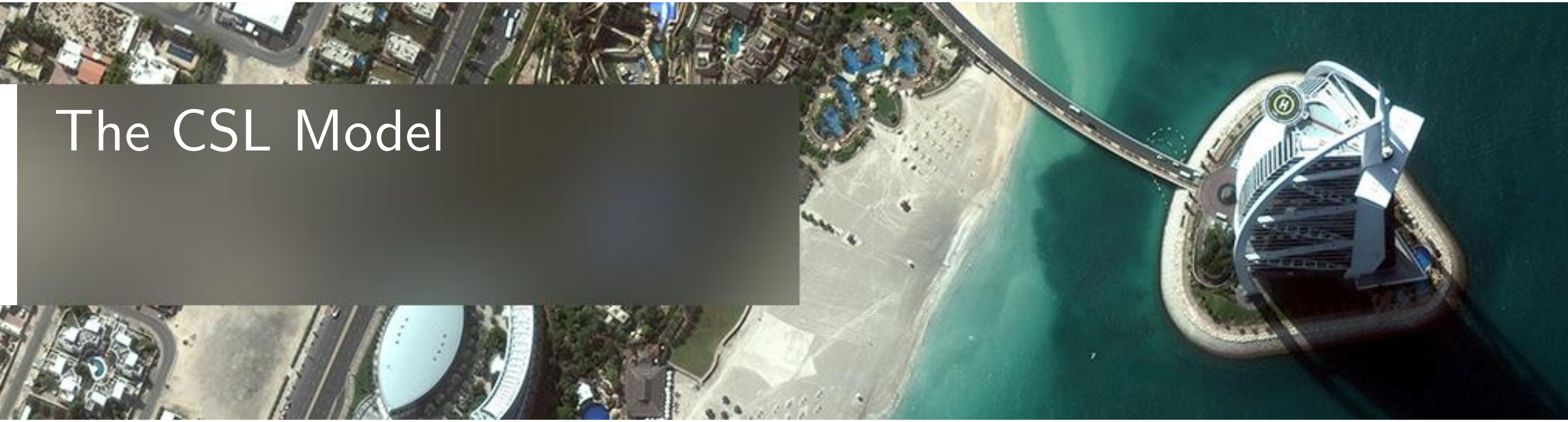
Definition 4. *An area zone* $\zeta^A_{\lambda_a,\lambda_b}$ *is a gray-scale size decomposition of an image* f*, if for all levels* h *and* $\forall\ \lambda_a \leq \lambda_t < \lambda_b$*, the following two properties hold:*

$$\Gamma^A_{\lambda_t}(T_h(\zeta^A_{\lambda_a,\lambda_b}(f))) = T_h(\gamma^A_{\lambda_t}(f) - \gamma^A_{\lambda_b}(f)), \quad (13)$$

$$\Gamma^A_{\lambda_t}(T_h(\zeta^A_{\lambda_a,\lambda_b}(f) - \gamma^A_{\lambda_t}(\zeta^A_{\lambda_a,\lambda_b}(f)))) = \emptyset. \quad (14)$$

- The two properties allow to interactively split and merge zones as required.

The CSL Model



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The Characteristic-Saliency-Level (CSL) Model

- The DAP vector field is a $2 \times (I-1) \times \texttt{ImageSize}$ volume set.
- In remote sensing images tend to be in the order of GBs.
- A detailed analysis, i.e. with $I > 128$ requires a prohibitively large amount of memory to store the DAP vector field.
- The CSL model is a compression model of DAP VF to a three band layer of the most relevant features.

The Characteristic-Saliency-Level (CSL) Model

- The extreme values of an entry in any of the two response vectors are 0 and $h_{\max}$ with the latter being the maximal image intensity value.
- Consider a point $x \in E$ and let $\check{dh}_{\gamma_\lambda^\Lambda}$ be the highest value in the positive response vector of DAP(x), i.e.:

$$\check{dh}_{\gamma_\lambda^\Lambda}(x) = \vee \Delta^{\Pi}(\gamma_\lambda^\Lambda(f))(x), \tag{15}$$

  which is given at a scale i.
- Note that i may not be necessary unique, i.e. the same response can be observed at other scales too.

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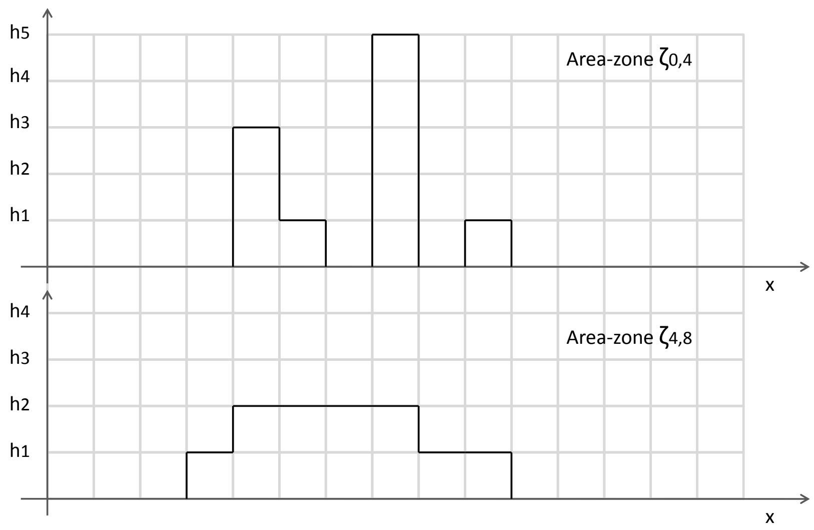
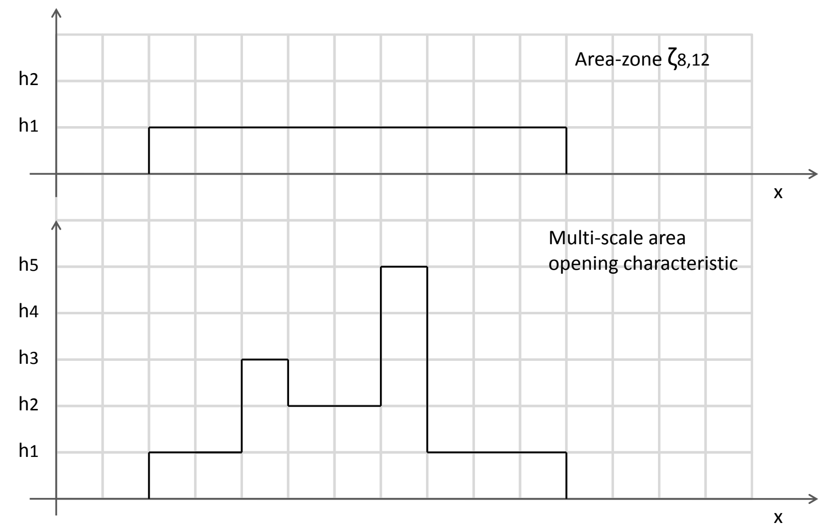
The Characteristic-Saliency-Level (CSL) Model



zone 1 and 2.



zone 3 and the characteristic responses

Example of area zone decomposition with $\vec{\lambda} = [0, 4, 8, 12]$.

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The Characteristic-Saliency-Level (CSL) Model

- However, since the most relevant information of the decomposition are contained within planes associated with smaller scales, the parameter:

$$\hat{i}_{\gamma^{\Lambda}_{\lambda}}(x) = \wedge i : (\gamma^{\Lambda}_{\lambda_{i-1}} - \gamma^{\Lambda}_{\lambda_i})(x) = \check{dh}_{\gamma^{\Lambda}_{\lambda}}(x), \tag{16}$$

is identified. That is, $\hat{i}_{\gamma^{\Lambda}_{\lambda}}$ is the smallest scale at which the positive response vector of the DAP(x) registers the maximal response.

- The equivalent scale parameter for the negative response vector is defined analogously:

$$\hat{i}_{\phi^{\Lambda}_{\lambda}}(x) = \wedge i : (\phi^{\Lambda}_{\lambda_i} - \phi^{\Lambda}_{\lambda_{i-1}})(x) = \check{dh}_{\phi^{\Lambda}_{\lambda}}(x). \tag{17}$$

- They are referred to as the *multi-scale opening* and *closing characteristic* respectively; *MOC* and *MCC*.

The Characteristic-Saliency-Level (CSL) Model

- Consider the following labeling scheme based on the maxima of the two response vectors:

  1. **convex**, if $\check{dh}_{\gamma^{\Lambda}_{\lambda}}(x) > \check{dh}_{\phi^{\Lambda}_{\lambda}}(x)$,
  2. **concave**, if $\check{dh}_{\gamma^{\Lambda}_{\lambda}}(x) < \check{dh}_{\phi^{\Lambda}_{\lambda}}(x)$,
  3. **flat**, if $\check{dh}_{\gamma^{\Lambda}_{\lambda}}(x) = \check{dh}_{\phi^{\Lambda}_{\lambda}}(x)$.

- This is the equivalent taxonomy to the ones proposed based on the differential morphological and area profile decompositions.
- The set of labels is referred to as the local curvature of the gray level function surface, where the attribute value determines the local spatial domain.

The Characteristic-Saliency-Level (CSL) Model

An intuitive multi-band segmentation scheme is then defined as follows:

$$\bar{i}(x) = \begin{cases} \hat{i}_{\gamma_\lambda^\Lambda}(x) & \text{if } \textbf{convex}, \quad (18a) \\ \hat{i}_{\phi_\lambda^\Lambda}(x) & \text{if } \textbf{concave}, \quad (18b) \\ 0 & \text{if } \textbf{flat}. \quad (18c) \end{cases}$$

This multi-level segmentation scheme can be further simplified to a 3-band segmentation by replacing the scale parameter with a fixed gray-level.

- The winning scale from the comparison in (18) is referred to as the **characteristic scale** or simply **characteristic** C.
- The response associated to the winning scale is called the **saliency** S and is denoted by $d\overline{h}$.

The Characteristic-Saliency-Level (CSL) Model

- The two, complemented by the **level** L of the pixel x after the iteration of the respective attribute filter at the reference scale i, denoted as $\hbar$, constitute the three bands of the CSL model.

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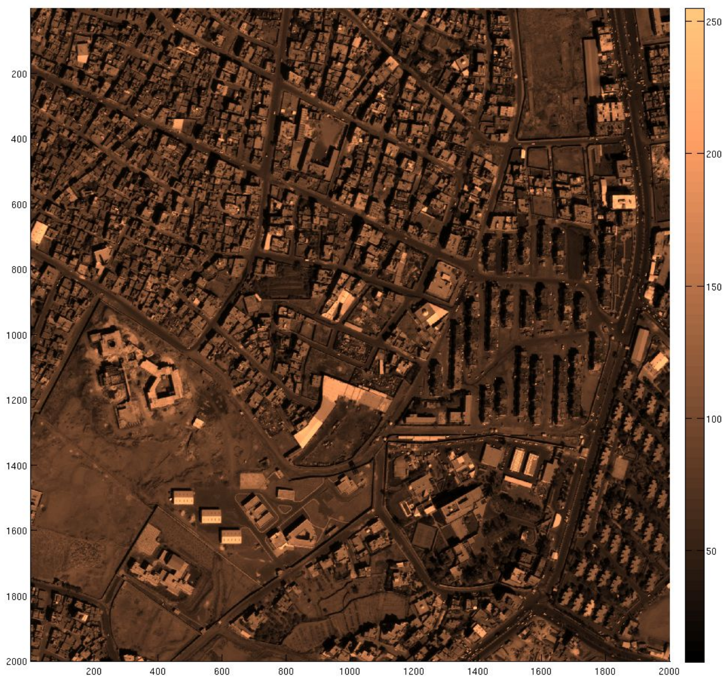
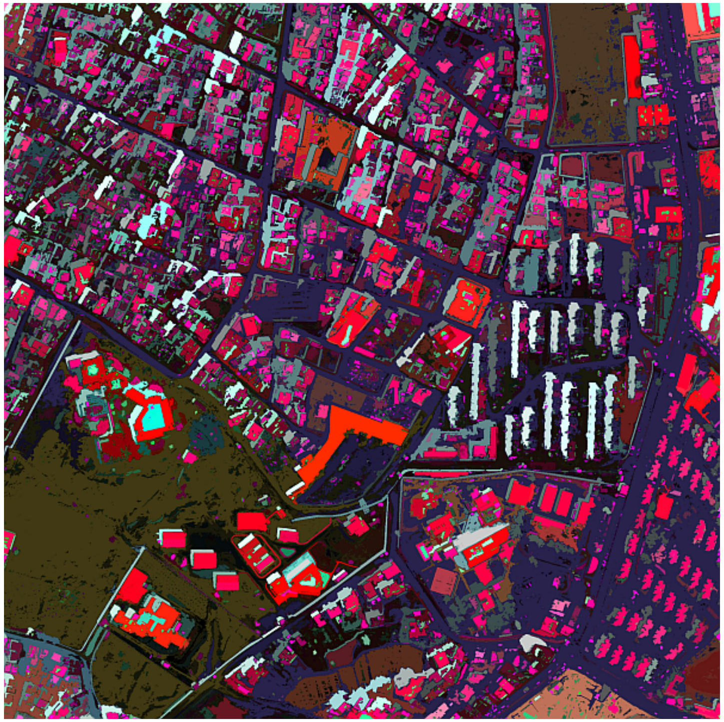
The Characteristic-Saliency-Level (CSL) Model

The 3 CSL bands can be fused using a non-linear mixture model to an HSV color-space representation. This is used for urban-pattern visualization and an example is shown.

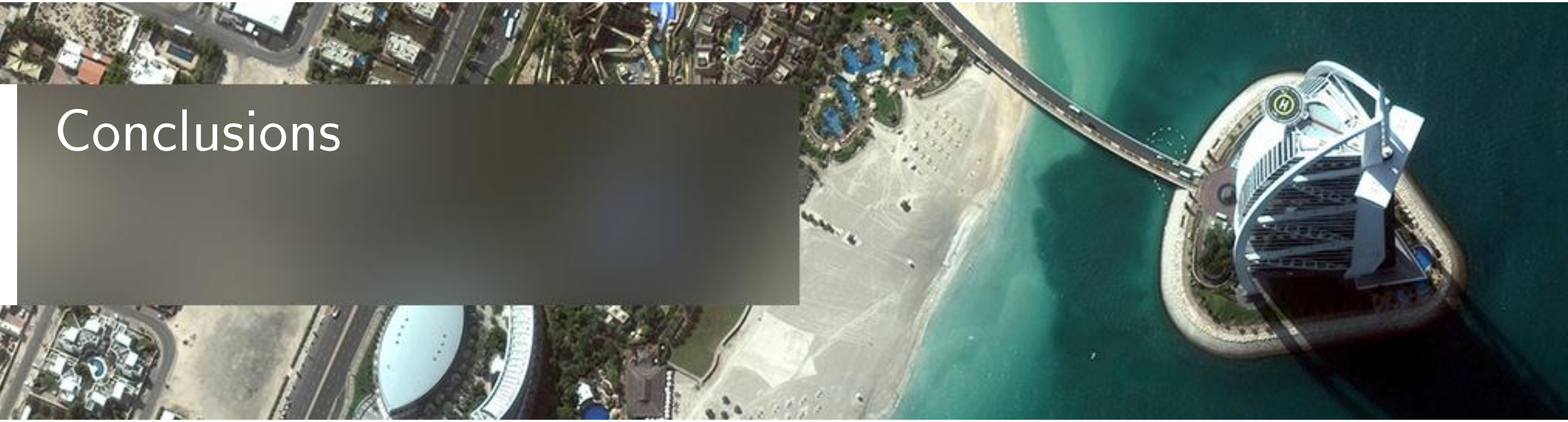


The input image of Sana'a, Yemen.



The CSL2RGB transform.

Conclusions



Conclusions

- Connected morphological operators can provide robust, efficient and very fast solutions to problems of a very demanding nature.
- For applications like the GHSL population its is only connected morphology that has so far offered solutions.
- The DAP vector fields and the CSL model are two powerful morphological methods for multi-scale analysis and compression respectively.
- The attribute zone decomposition can be computed using the Max-Tree algorithm, which is a matured technology with many advantages.

THANK YOU

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