

# Graph-based features for texture discrimination

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## Abstract

*Graph-based features, such as the number of connected components, edges of a given orientation and vertices per unit area, and the number of vertices and pixels per connected component, are proposed for the analysis of textures which consist of structural elements. The proposed set of features is compared with features obtained by a typical filter-based scheme which makes use of Gabor filters. The discrimination properties of the two types of features are assessed by evaluating the separability of sets of feature vectors which are derived from different types of texture using the Mahalanobis distance. The graph-based features are shown to be superior to the filter-based features for the class of concerned textures. They are particularly suited for discrimination between textures which have the same spatial and orientation regularity but consist of elements of different form.*

**Keywords:** texture features, graph representation, discrimination, Mahalanobis distance, Gabor filters.

## 1. Introduction

Many texture feature extraction operators consist of linear filtering followed by a non-linear point operation and derivation of local statistics [3], [7], [11]. While the linear step aims at extracting feature values which are different for different types of texture, the computation of local statistics, such as averaging, is aimed at making the feature values which are obtained for different points of one and the same texture type more similar to each other.

For a specific class of texture which consist of structural elements like the ones shown in Figure 1, this approach does not give the best results. For such textures, we propose features such as the number of regions, edges of a given orientation and corners per unit

area, and the number of corners and pixels per region. These features are related to a graph-based representation of the underlying texture.

The concerned set of features is compared with features obtained by a typical filter-based scheme which makes use of Gabor filters. The discrimination properties of the two types of features are assessed by evaluating the separability of sets of feature vectors which are derived from different types of texture using the Mahalanobis distance [4], [5].

The rest of the paper is organized as follows: Section 2 presents the set of graph-based features. Section 3 gives results regarding the separability of clusters of feature vectors obtained from different textures. The concerned set of features is compared to a set of Gabor energy features. Finally, in Section 4, some conclusions are drawn.

## 2. Feature extraction

The features described below are obtained for each pixel of an image using a rectangular window of unit area with a center at the concerned pixel. The area unit is taken to be one quarter of the image size. We propose the following features:

**Number of connected components per unit area.** The textures considered in this study consist of polygons which can be viewed as connected components of the graph which is comprised of the edges and vertices of all polygons in the texture image. This graph can be decomposed in disjoint subgraphs, one such subgraph per polygon. We take the number of such connected components per unit area of the image as one of the features. Note that the concerned polygons are also connected components with respect to the uniform gray level value of the pixels within each region. Therefore, the value of this feature can be easily obtained by applying a connected components analysis at the pixel level.

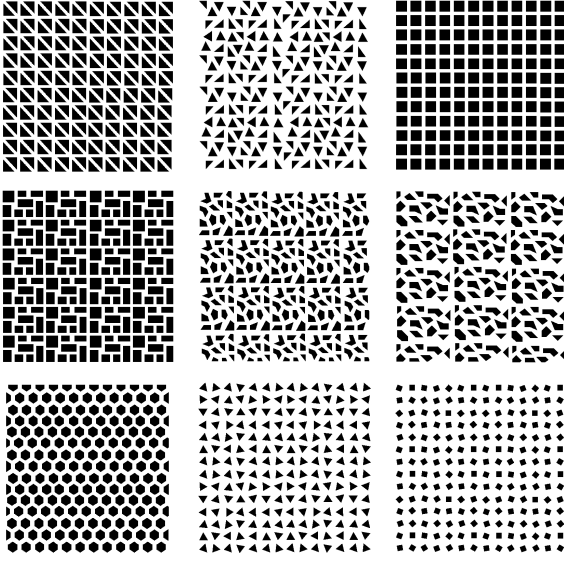


Figure 1. Test textures denoted by T1 to T9 (from top-left to bottom-right, respectively).

**Number of edges of a given orientation per unit area.** This feature is characteristic of an attributed graph in which each graph edge represents an edge of a polygon with a certain orientation. The number of edges with a given orientation per unit area is a feature which can be quite effective in discrimination tasks: for instance, while the edges of the graphs derived from texture types T3 and T4 are characterized by two orientation values only, texture type T1 is characterized by three edge orientation values, and T8 and T9 by 10 different orientation values.

The graph edges with a given orientation are determined by detecting the corresponding polygon edges. A number of authors have proposed oriented edge detector operators based on local oriented energy derived with a quadrature-phase pair of filters [1], [2], [10]. In this study, we use a pair of even (e) and odd (o) Gabor filter kernels:

$$g_{\gamma,\sigma,\theta,\lambda}^{(e)}(x,y) = e^{-\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2}} \cos\left(2\pi \frac{x'}{\lambda}\right) \quad (1)$$

$$g_{\gamma,\sigma,\theta,\lambda}^{(o)}(x,y) = e^{-\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2}} \sin\left(2\pi \frac{x'}{\lambda}\right) \quad (2)$$

where

$$\begin{aligned} x' &= x \cos \theta - y \sin \theta \\ y' &= x \sin \theta + y \cos \theta \end{aligned}$$

The local oriented energy associated with the above mentioned filters is defined as follows:

$$E_{\gamma,\sigma,\theta,\lambda}(x,y) = \sqrt{(I * g_{\gamma,\sigma,\theta,\lambda}^{(e)})^2 + (I * g_{\gamma,\sigma,\theta,\lambda}^{(o)})^2} \quad (3)$$

where  $I$  denotes the input image,  $*$  the convolution operation,  $(x,y)$  are the image coordinates, and  $\gamma, \sigma, \theta$  and  $\lambda$  are parameters.

We choose the value of the parameter  $\gamma$ , called the spatial aspect ratio, to be  $\gamma = 3$ . The value of the parameter  $\sigma$  which determines the spatial extent of the convolution kernels is chosen to be  $\sigma = 2.5$  for images of size  $512 \times 512$ . The value of the ratio  $\sigma/\lambda$  determines the spatial frequency bandwidth of the filters; we take  $\sigma/\lambda = 0.56$ , a value which corresponds to a half-response bandwidth of one octave.

In order to eliminate the suboptimal response for adjacent orientations, a ‘winner takes all’ orientation competition mechanism [9] across all orientations is applied in each point:

$$\hat{E}_{\gamma,\sigma,\theta_k,\lambda}(x,y) =$$

$$\begin{cases} E_{\gamma,\sigma,\theta_k,\lambda}(x,y), & \text{if } E_{\gamma,\sigma,\theta_k,\lambda}(x,y) = \max_i \{E_{\gamma,\sigma,\theta_i,\lambda}(x,y)\} \\ 0, & \text{if } E_{\gamma,\sigma,\theta_k,\lambda}(x,y) < \max_i \{E_{\gamma,\sigma,\theta_i,\lambda}(x,y)\} \end{cases} \quad (4)$$

$$i, k \in \{0, 1, \dots, N-1\}, \theta_k = \frac{\pi * k}{N}, N = 8.$$

The resulting  $N$  ( $N = 8$ ) oriented energy images are binarized by thresholding at 65% of the maximum value range.

The number of edges with a given orientation per unit area is obtained by applying a connected components analysis to the corresponding binary images.  $N$  features are extracted in this way, one for each of the  $N$  discrete orientation values.

**Number of vertices per unit area.** The vertex detection is based on a local energy model [10] using the same quadrature-phase pair of Gabor filters.

An energy image of enhanced vertex areas is obtained by computing the convolution of an edge energy image  $E_{\gamma,\sigma,\theta,\lambda}(x,y)$  with a Gabor filter kernel of orientation which is orthogonal to the one used for obtaining the edge energy image, and averaging this new energy over all orientations:

$$V_{\gamma,\sigma,\lambda}(x,y) = \frac{1}{N} \sum_{\theta_k=0}^{N-1} \sqrt{(E_{\gamma,\sigma,\theta_k,\lambda} * g_{\gamma,\sigma,\theta_k+\frac{\pi}{2},\lambda}^{(e)})^2 + (E_{\gamma,\sigma,\theta_k,\lambda} * g_{\gamma,\sigma,\theta_k+\frac{\pi}{2},\lambda}^{(o)})^2}$$

(The parameters have the same values but  $\gamma, \gamma = 1.0$ )

The resulting image is binarized by thresholding at 65% of the maximum value range.

**Number of vertices per connected component.** This feature is related to the number of vertices per unit area and the number of connected components per unit area, but it can give better results in a number of situations. For instance, the relative difference in the values of this feature for texture types T2 and T3 is larger than the relative differences which can be obtained using the other two features separately.

Although this feature can be computed as the ratio between the number of vertices per unit area and number of connected components per unit area, we derive it by counting separately the number of vertices of each polygon. Next to giving the opportunity of extracting more precise values by means of excluding polygons which intersect with the border of the analysis window, this approach gives the possibility of extending this feature from a scalar value to a vector which represents a histogram of the number of vertices per connected component.

**Number of pixels per connected component.** In graph theoretic sense, this feature characterizes a measure associated with graph faces which correspond to the polygons in the input images. It can be quite effective in discrimination tasks: for instance, the value of this feature for texture T3 is quite different from its value for texture T9. We derive this feature by applying a connected components analysis at pixel level directly on the input images and by counting the number of pixels in each of the connected components. This feature can also be extended from a scalar to a vector to represent the histogram of number of pixels per connected component.

Table 1 summarizes the correspondence between the structural descriptors described above and the corresponding graph concepts.

Structural descriptors	Graph concepts
Polygon regions	Disjoint graphs
Oriented polygon edges	Attributed graph edges
Polygon vertices	Graph vertices
Vertices per polygon	Vertices per connected component
Pixels per region	Face attribute

**Table 1. Correspondence between structural descriptors and graph concepts.**

### 3. Feature separability comparison

A 12-dimensional feature vector which has as components the features described above is assigned to each pixel in the input image. The feature vectors computed in different points of a texture image form a cluster in

the multidimensional feature space. For classification and segmentation purposes it is necessary that clusters which correspond to different textures be separable. The larger the separability, the better the results.

We assess the cluster separability by measuring the Mahalanobis distance [5], [8] for every pair of the nine test textures shown in Figure 1. In order to build a cluster, we selected one thousand vectors at random positions from each texture image. Table 2 gives the values of the Mahalanobis distance between pairs of clusters of feature vectors in the case of the graph-based features.

The separability of graph-based feature vector clusters is next compared with the separability of feature vector clusters in a 12-dimensional feature space derived by local Gabor energy computation [3], [6].

A 12-dimensional Gabor energy feature vector is assigned to each pixel in the image. It comprises the values of the local Gabor energy (3) for 6 orientations and 2 scales:  $\theta_k = \pi * k/N$  with  $k \in \{0, 1, \dots, N\}$ ,  $N = 6$ ;  $\sigma/\lambda = 0.56$ ,  $\sigma \in \{2.5, 5\}$ ,  $\gamma = 0.5$ . As a post-processing operation, an averaging with a rectangular window of the same size as the window size in the graph-based approach (one quarter of the image) is performed. Again, a cluster of one thousand feature vectors taken at random positions is built for each image. Table 3 shows the values of the Mahalanobis distance between pairs of clusters for this second case. Speaking in terms of misclassification probability, two clusters which have normal distributions with the same standard deviation and a Mahalanobis distance of 5.31, which is the minimum distance obtained in the graph-based approach, overlap for less than  $0.34 * 10^{-5}$ . By contrast, the minimum distance between clusters of Gabor energy feature vectors is 1.76, which corresponds to an overlap of 18.4%.

One can observe from the two tables that, for any pair from the considered set of textures, the graph-based features give better separability than the filter-based features. Although for some pair of textures such as T1 and T7, the Gabor energy scheme leads to a quite good separability due to the spatial and orientation regularity of texture elements, the proposed set of graph-based features is more descriptive and performs better in all situations. We emphasize the better separability results especially for cases where the texture elements exhibit a higher degree of irregularity (see, for example, the pair T2 and T5). Gabor energy features lead to a very small separability for textures which have the same regular spatial arrangement of elements and are composed of different, rather small structural elements, such as T8 and T9. In such cases, the proposed set of graph-based features offers better discrimination

	T1	T2	T3	T4	T5	T6	T7	T8	T9
T1	-	7.82	8.12	12.71	7.60	7.13	9.06	12.03	13.55
T2		-	9.30	9.07	13.79	5.49	10.76	7.22	10.88
T3			-	5.31	8.38	8.04	13.50	17.65	14.53
T4				-	8.62	9.43	13.14	18.62	17.10
T5					-	10.94	17.98	15.73	10.27
T6						-	12.95	10.61	9.12
T7							-	18.81	18.70
T8								-	9.60
T9									-

Min. 5.31, Max. 18.81, Avg. 11.48.

**Table 2. Mahalanobis distance values between pairs of clusters of graph-based feature vectors.**

	T1	T2	T3	T4	T5	T6	T7	T8	T9
T1	-	5.78	5.24	5.07	3.55	4.08	8.24	8.55	8.40
T2		-	7.75	6.26	3.33	3.44	7.69	3.06	3.67
T3			-	1.76	4.71	4.90	10.32	13.14	10.51
T4				-	4.79	5.26	10.47	11.45	9.90
T5					-	2.66	5.23	2.91	4.40
T6						-	5.68	3.02	2.92
T7							-	7.49	10.22
T8								-	2.67
T9									-

Min.: 1.76, Max.: 13.14, Avg.: 6.24.

**Table 3. Mahalanobis distance values between pairs of clusters of feature vectors obtained with local Gabor energy.**

possibilities.

#### 4. Summary and conclusions

In this paper we proposed a set of graph-based features related to characteristics of polygonal texture elements. These features are: the number of connected components (regions) per unit area, a histogram of the number of attributed (oriented) edges per unit area for eight different orientations, the number of vertices (corners) per unit area, the number of vertices per connected component (region) and the average size of a face of a connected component (region). The graph-based features were compared with Gabor energy features regarding the separability of clusters of feature vectors obtained from different textures, and it was shown that the former features are superior to later ones for the class of considered textures. From the experiments done one can observe that graph-based features are particularly suited for discrimination between textures which have the same spatial and orientation regularity but consist of texture elements of different form.

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