Texture analysis using Rényi's generalized entropies

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Abstract

We propose a texture analysis method based on Rényi's generalized entropies. The method aims at identifying texels in regular textures by searching for the smallest window through which the minimum number of different visual patterns is observed when moving the window over a given texture. The results show that any of Rényi's entropies can be used for texel identification. However, the second order entropy, due to its robust estimation, is the most reliable. The main advantages of the proposed method are its robustness and its flexibility. We illustrate the usefulness and the effectiveness of the method in a texture synthesis application.

Rényi's generalized entropies

Rényi's entropy family is one of the most popular generalizations of Shannon's entropy.

Definition 1 *If a random variable ξ takes the values $[x_i]_{i=1\ldots N}$ with probabilities $[p_i = P(\xi = x_i)]_{i=1\ldots N}$, then the generalized entropy of order q of ξ is defined as*

$$H_q = \begin{cases} \dfrac{\log \sum_{i=1}^{N} p_i^q}{1-q} & \text{for } q \neq 1, \\ \lim_{q\to 1} \left(\dfrac{\log \sum_{i=1}^{N} p_i^q}{1-q} \right) = -\sum_{i=1}^{N} p_i \log p_i & \text{for } q = 1, \\ -\log \max_{i=1\ldots N} (p_i) & \text{for } q \to \infty. \end{cases} \qquad (1)$$

- H_0 counts the number of values x_i for which p_i is nonzero.
- H_1 is the Shannon entropy.
- H_2 is called the quadratic entropy.
- H_∞ is called the min-entropy.
- High order entropies give more weight to highly probable events, while minimizing the contribution of the rarely occurring events – the outliers.
- $H_q = 0$ if and only if ξ takes one single value.
- H_q is maximum when ξ can take any of the N values with equal probability. In this case, $H_q = \log N$.
- For the same PDF, H_q is a decreasing function of q.
- H_q measures the amount of uncertainty in predicting the output of a probabilistic event: when the uncertainty is reduced, H_q decreases.

Rényi's generalized entropies of texture

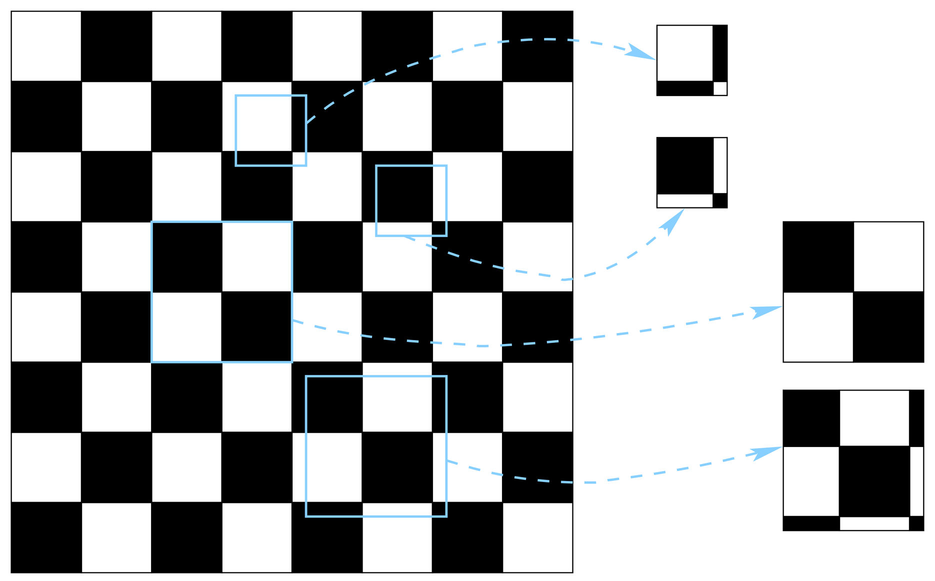

Figure 1: *A synthetic texture and the patterns that can be seen in this texture when looking through square windows of various sizes. The white and black checkers in the synthetic texture are $k \times k$ pixels wide. A pattern seen through a given window at a given position is considered a visual event. Two observed patterns are considered as the same visual event if they are circularly shifted versions of each other. With this rule, one can compute the probability of occurrence of a given visual event in the considered texture. From the probabilities of occurrence of all visual events one can compute the generalized entropies of the texture as functions of the window size.*

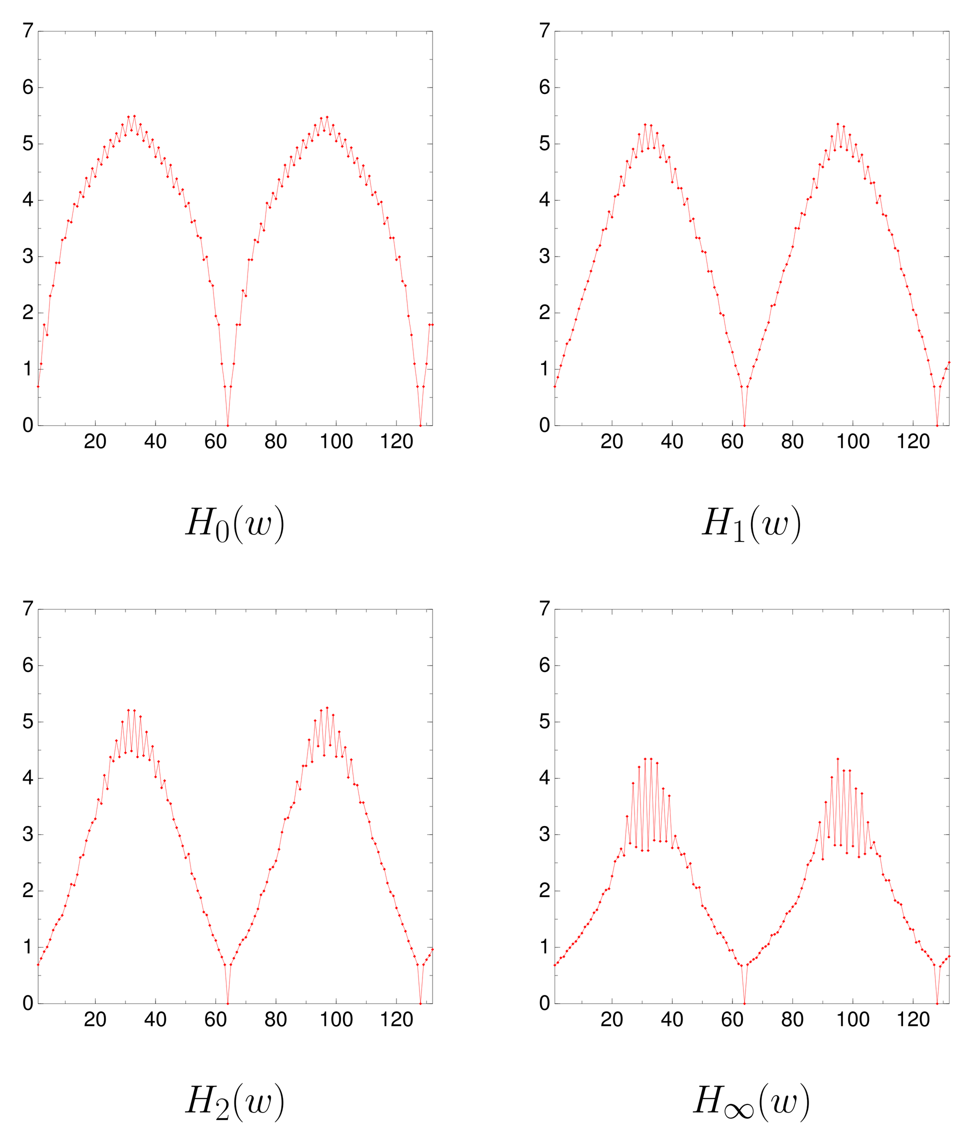

Figure 2: *Rényi's entropies of the texture in Fig. 1, for $k = 32$, when using square observation windows. The values on the abscissa represent the window size w, while the values on the ordinate represent $H_q(w)$. All entropies reach their minima for window sizes that are multiples of $2k \times 2k$.*

Rényi's entropies of natural textures

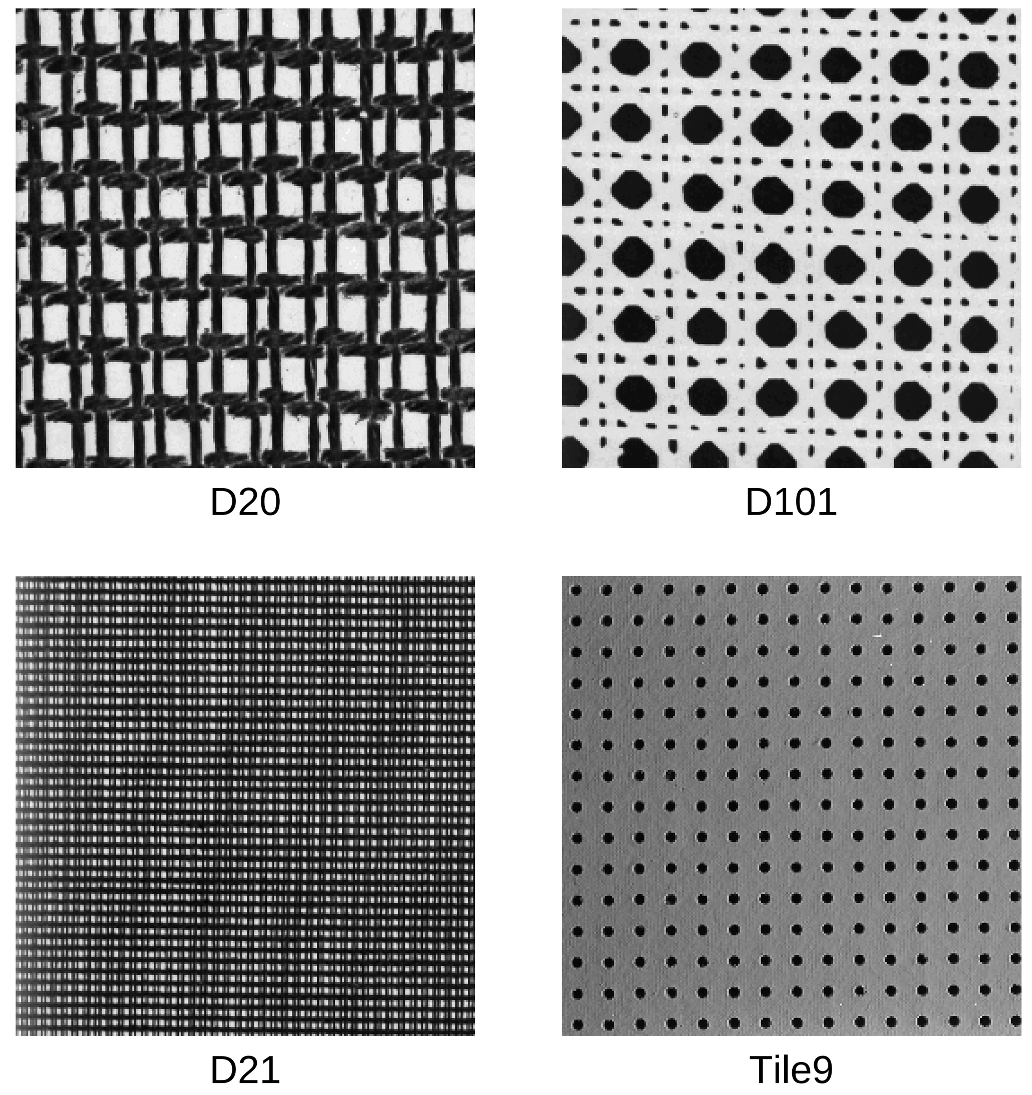

Figure 3: *Natural textures consisting of square shaped texels.*

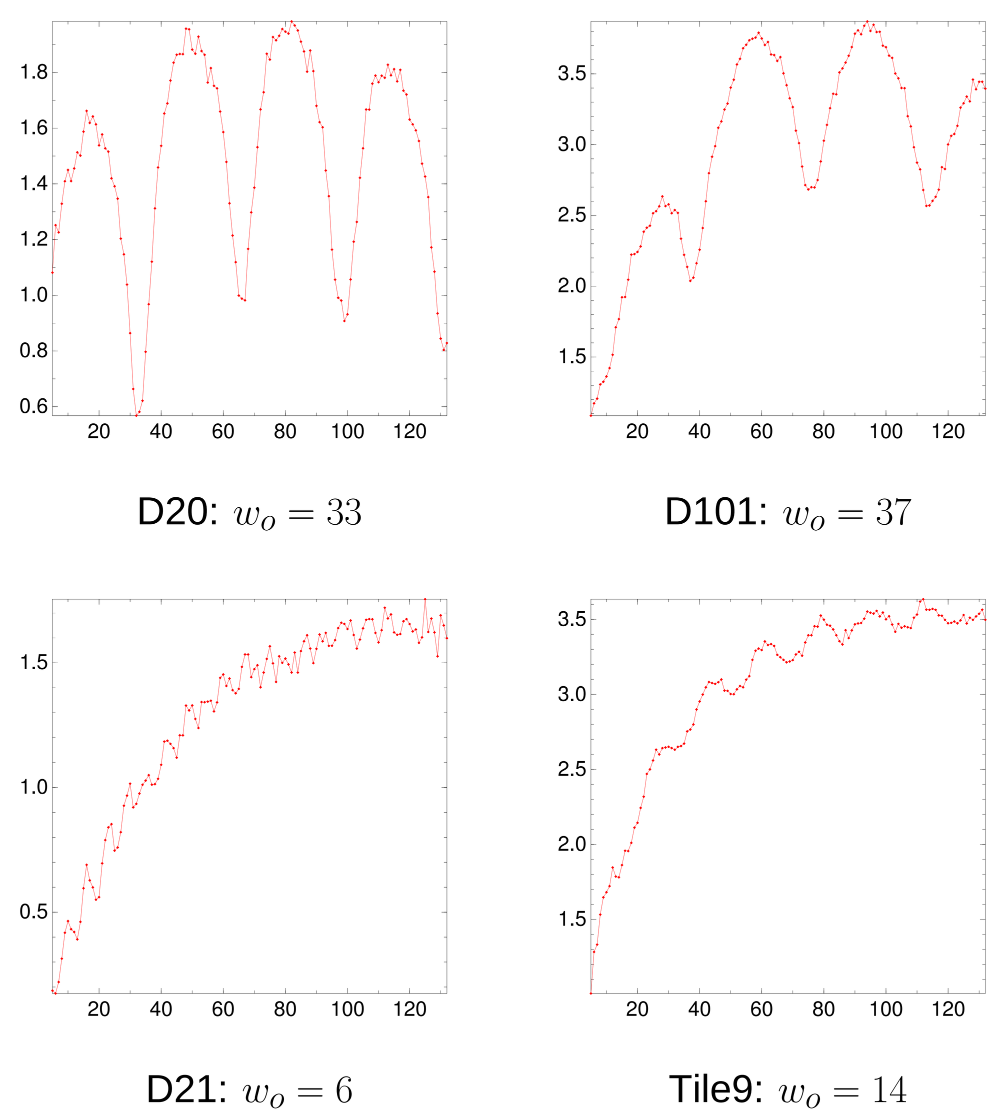

Figure 4: *Rényi's quadratic entropies of the textures in Fig. 3. The values on the abscissa represent the window size w, while the values on the ordinate represent the entropy $H_2(w)$. The caption of each plot gives the texture name and the smallest window size for which $H_q(w)$ has a local minimum (i.e. $H_q(w) \leq H_q(w-1)$ and $H_q(w) \leq H_q(w+1)$).*

Computational aspects

- For deciding whether two patterns represent the same visual event we compare their histograms.
- The histogram comparison is done by means of the Kolmogorov-Smirnov statistical test at a significance level 0.05.
- In the computation of the quadratic entropy we use the algorithm described in [**?**].

Application to texture synthesis

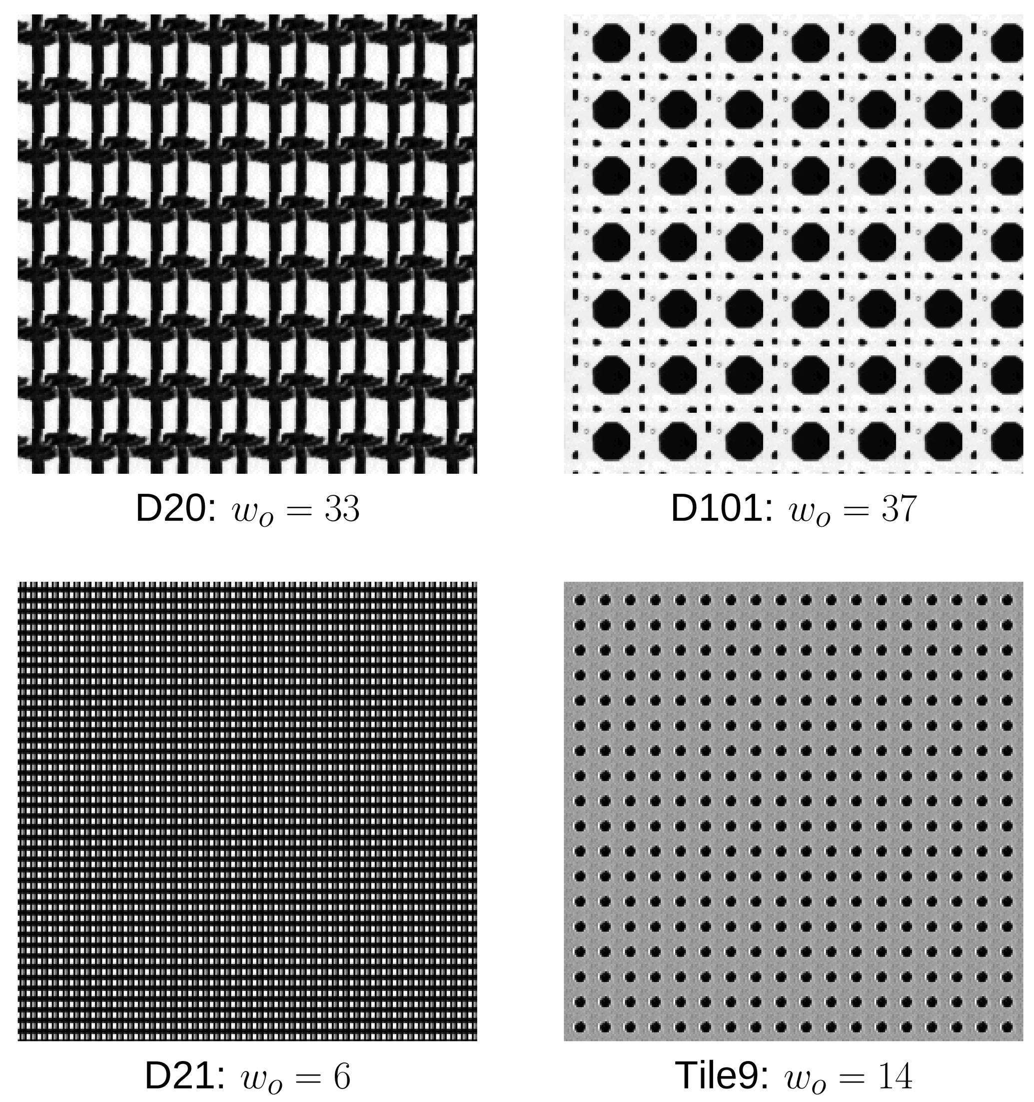

Figure 5: *The synthesized counterparts of the texture in Fig. 3. The caption of each image gives the size of the texel used in the synthesis of that texture.*

Conclusions

- The proposed texel identification method is robust and flexible.
- It is superior to previous methods for structural texture analysis that use co-occurrence matrices, Fourier analysis, or autocorrelation functions.

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