Exercises for

Chapter 6: Vector Visualization

1 EXERCISE 1

Consider a scalar field $f(x, y) \rightarrow \mathbb{R}$. The gradient ∇f is a vector field, so we can compute its vorticity, $\mathbf{v} = \text{rot}(\nabla f)$. What is the value of $\mathbf{v}$?

Hints: Use the definitions of the gradient and vorticity based on partial derivatives of f.

2 EXERCISE 2

Consider now a 2D vector field $\mathbf{v}(x, y)$, and its vorticity rot $\mathbf{v}(x, y)$. We now compute the divergence field $d = \text{div}(\text{rot}\ \mathbf{v})$. What is the value of d?

Hints: Use the definitions of divergence and vorticity based on partial derivatives of a function of two variables.

3 EXERCISE 3

Consider the following three 3D vector fields

$$\mathbf{v}_1(x, y, z) = (-y, x, 0) \tag{3.1}$$
$$\mathbf{v}_2(x, y, z) = (y, x, 0) \tag{3.2}$$
$$\mathbf{v}_3(x, y, z) = (3x, 3y, 0) \tag{3.3}$$

Plot these three vector fields using hedgehog glyphs (oriented arrows) for a square domain centered at the origin and embedded in the xy plane.

4 EXERCISE 4

Consider the following three 3D vector fields (the same ones as for Exercise 3):

$$\mathbf{v}_1(x, y, z) = (-y, x, 0) \tag{4.1}$$
$$\mathbf{v}_2(x, y, z) = (y, x, 0) \tag{4.2}$$
$$\mathbf{v}_3(x, y, z) = (3x, 3y, 0) \tag{4.3}$$

Which of these three fields has a zero curl? Which has a zero divergence? Argue your answer by computing the actual divergence and curl values.

5 EXERCISE 5

Vector field visualization is considered to be, in general, a more challenging problem than scalar visualization. Indeed, intuitively speaking, for a sampled dataset of, say, N points, a scalar field would have N values, whereas a (3D) vector field would have $3N$ values to show. Consider now the case of visualizing a scalar field with N samples *vs* visualizing a 3D vector field with $N/3$ samples. Both fields need to store the same amount of data values, *i.e.*, N. Which of the following assertions do you support for this situation:

- Visualizing both fields is, in general, equally challenging
- Visualizing the vector field is, in general, more challenging than visualizing the vector field.

Support your answer with a detailed explanation.

6 EXERCISE 6

Consider a smooth 2D scalar field $f(x, y)$, and its gradient ∇f, which is a 2D vector field. Consider now that we are densely seeding the domain of f and trace streamlines in ∇f, upstream and downstream. Where do such streamlines meet? Can you give an analytic definition of these meeting points in terms of values of the scalar field f?

Hints: Consider the direction in which the gradient of a scalar field points.

7 EXERCISE 7

Consider a smooth 2D scalar field $f(x, y)$, and its gradient ∇f, which is a 2D vector field. Consider now the divergence $d = \text{div}(\nabla f)$, also called the Laplacian of f. What is the relation between the local extrema (minima and maxima) of f and those of d?

- The minima of d are the maxima of f
- The maxima of d are the minima of f
- Both the above are true
- None of the above are true

Hints: Consider the relationship between minima and maxima of divergence and so-called *sinks* and *sources* of a vector field.

8 EXERCISE 8

Can any 2D vector field $\mathbf{v}(x, y)$ be seen as the gradient of some scalar field $f(x, y)$? That is: Given any vector field $\mathbf{v}$, can we find a scalar field f, so that $\mathbf{v}(x, y) = \nabla f(x, y)$, for all points (x, y)? If so, argue why, and show how we construct f from $\mathbf{v}$. If not, show and discuss a counter-example.

Hints: Consider the vector calculus identities involving curl and gradient.

9 EXERCISE 9

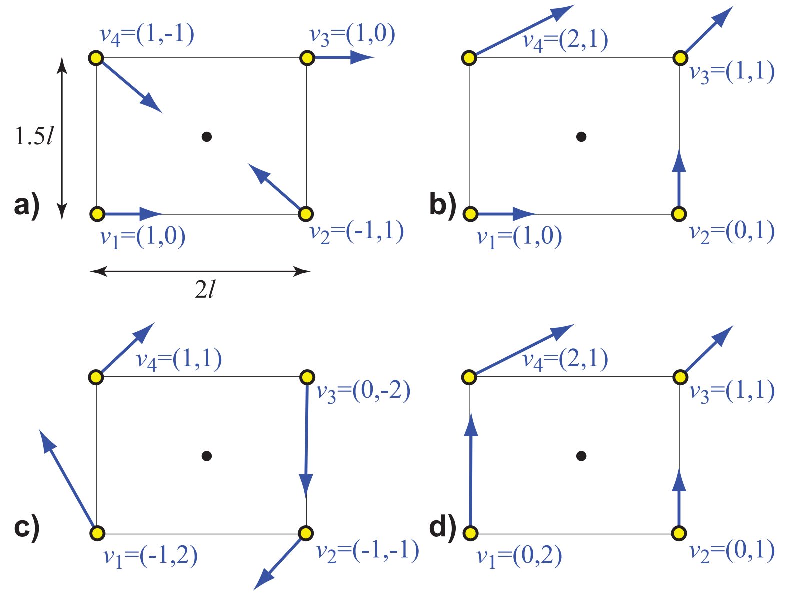
Intuitively, we can think of the (mean) curvature of an oriented 3D surface as a scalar function, defined on the surface, which takes large positive values where the surface is convex, large negative values where the surface is concave, and is zero where the surface is locally flat. How can we compute such a scalar curvature function using just the vector-field defined by the surface normals?

10 EXERCISE 10

Vector glyphs are one of the simplest, and most used, methods for visualizing vector fields. However, careless use of vector glyphs can lead to either visual clutter (too many glyphs drawn over the same small screen space) or visual subsampling (large areas in the field's domain which do not contain any vector glyph). Given a 3D vector field which we want to visualize with vector glyphs, describe all parameters that one can control, and how these should be controlled, to reduce both visual clutter and visual subsampling.

11 EXERCISE 11

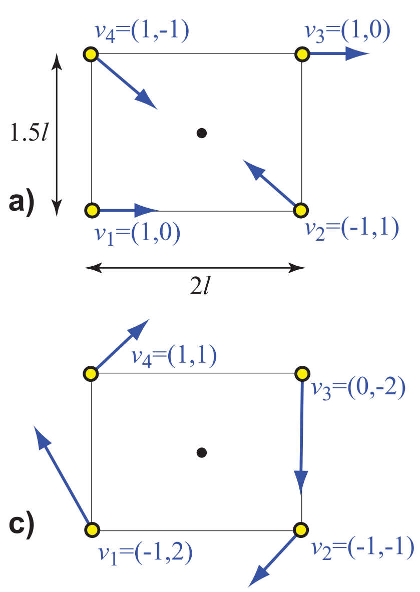
Consider the rectangular areas shown in the figure below. All areas have the same aspect ratio $(2l, 1.5l)$. For each case, we define four 2D vector values $\mathbf{v}_1, \ldots, \mathbf{v}_4$ at the area's vertices, and visualize these by the blue vector glyphs. Now, for each area, look at its center (indicated by a black point). What is the value of the vector field that you perceive there, based on the surrounding four blue glyphs? Draw a vector glyph at the central point to indicate the direction and magnitude of your perceived vector.



What is the vector value you perceive at the center of each area, given the vertex values?

12 EXERCISE 12

Consider the figure from the previous exercise (shown again below). For each area, compute the value of the vector field at the central black point, assuming that we are using bilinear interpolation of the four vertex values. Next, draw the resulting interpolated vectors using vector glyphs at the respective central points. Finally, compare the result with the one of the previous exercise. Do you see any differences? If so, which ones, and how do you explain them?



What is the vector value produced by bilinear interpolation at the areas' centers, given the vertex values?

13 EXERCISE 13

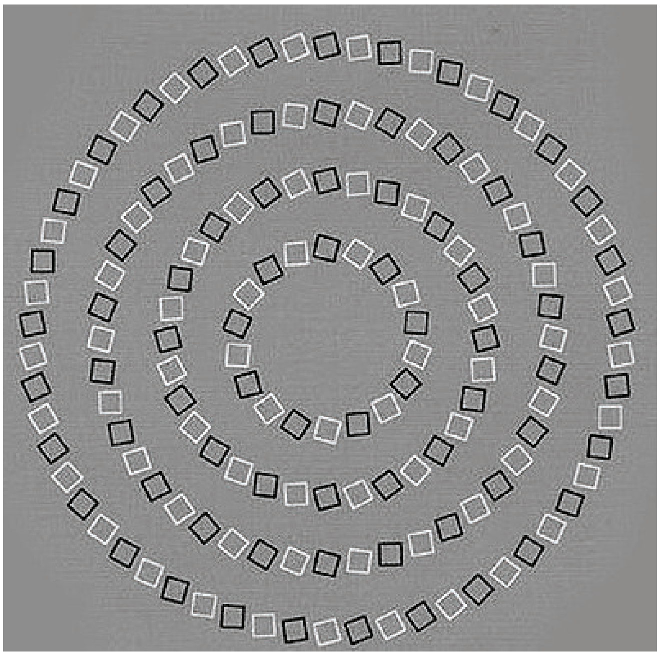
Consider the use of a 3D vector-glyph plot for the visualization of a 3D vector field. Each glyph is drawn as an arrow oriented and scaled by the vector field's direction and magnitude respectively. Without any additional cues such as interactively changing the viewpoint, it is clearly very hard to perceive the correct orientation of the resulting arrow glyphs in 3D. Propose and describe two visual enhancements (not using interaction) that would make the perception of 3D orientation of the resulting vector glyphs easier from a given viewpoint.

14 EXERCISE 14

Consider a 2D vector field defined over a 2D square domain. At each point $\mathbf{p}$, the vector field is $\mathbf{v}(\mathbf{p}) = \left(\frac{\mathbf{p}-\mathbf{c}}{\|\mathbf{p}-\mathbf{c}\|}\right)^{R\left(\frac{\pi}{2}\right)}$. Here, $\mathbf{c}$ is the center of the square, and $R(\alpha)$ denotes a rotation by α degrees clockwise. We visualize this vector field with a special kind of glyph: At chosen sample points $\mathbf{p}$, we draw a square which has two edges parallel to the vector $(\mathbf{v})^{R(\beta)}$, where β is a small angle of about 20 degrees. A visualization using this technique is shown in the image below.

For this type of vector field and visualization, answer in detail the following questions:

- What are the streamlines of the vector field looking like?
- Looking at the figure below, what can you say about the vector field? Describe the vector field *as you see it in the figure* in a few sentences. Does this visualization accurately convey the vector field? If so, argue why. If not, argue what are the differences between the perceived vector field and the actual one.
- Which design elements of the glyph (color, shape, size, orientation, other) influence the strongest the perception of the vector field?



What type of vector field do you see depicted by the square glyphs?

15 EXERCISE 15

Visualizing time-dependent vector fields (either in the 2D or 3D case) is highly challenging. The main problem here is that, apart from the difficulty of showing the direction and magnitude of a vector field, we have to somehow show how these quantities change in time. One possible way to do this is to highlight spatial regions where these quantities significantly change at each time moment – that is, to compute a derivative of the vector field over time, and to overlay this information atop of an instantaneous vector field visualization generated by classical methods, such as e.g. vector glyphs, streamlines, or image-based techniques. In this context, propose a metric that would intuitively measure the change in time of the direction and/or magnitude of a vector field. Describe your metric analytically, and argue why it would be a good solution for the above goal of highlighting regions of rapid change.

16 EXERCISE 16

Simplified visualizations for vector fields reduce the problem of showing the vector data at each sample point in the field's domain to the problem of partitioning the domain into a set of regions, so that the field's variation over any such region can be easily represented by classical techniques such as vector glyphs or streamlines. For this to work, we need to know how to represent the 'average' vector field over some given spatial region R of a domain, where we know that our vector field has limited variation. Describe two visual representation techniques that can achieve this type of representation.

Hints:

- First, define what is the constraint of the vector field variation over a given region R
- Next, propose a way to represent the 'average' vector field over R, as well as the variation (deviation from the average) of the field over R

End of Exercises for
Chapter 6: Vector Visualization
